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A statistical evaluation of measurement error in determining moisture content in corn

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A statistical evaluation of measurement error

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by

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1. PROBLEM IDENTIFICATION

1.1. Introduction

Moisture content is a key quality characteristic in defining the grade of corn. If the moisture content of a load of corn presented for sale is higher than the acceptable trade standard, then the market value of this corn will be reduced, and its price will be discounted accordingly.

Quality characteristics, such as moisture content in corn, must be measured in order to assign the appropriate grade to a commodity. But, the grade assigned is only as accurate as the methods by which its quality is determined. That is, if a measuring device or method is unreliable, then the grade is less informative, less valuable, and less effective in serving its economic purpose.

Thus, the problem addressed in this thesis, that of measurement error in determining moisture content in corn, is considered a problem in the general area of grading.

1.2. Grading and its Economic Purpose

Grading is a method by which a commodity is classified according to its quality. Individual grades are defined with respect to certain physical attributes of the product that are considered to be indicative of its quality. Specified levels of these characteristics are grouped together into categories, and each category is assigned a grade in the form of a letter, number, or word. A set of grades, then, is a classification system designed to provide concise description of a product's quality.

As indicators of a commodity's quality, and therefore its value, grades are intended to serve an economic purpose. The basic objective of a grading system is to provide market information in terms of furnishing uniform description of a product's quality. This has impact on both the operational efficiency and the pricing efficiency of the market.

From the operational standpoint, grading

1) allows for selling by description, rather than by personal inspection;
2) reduces uncertainty between buyers and sellers, due to the standardization of the product by grade;
3) increases specialization in the use of the product with respect to particular quality characteristics;
4) reduces the expense of competitive brand advertising; and
5) allows for product differentiation, enabling consumers to identify the product quality that most satisfies their preferences.

In these ways, grading ultimately helps to reduce marketing costs by increasing the efficiency of the marketing process.

In terms of pricing efficiency, grading

1) allows for ease in communication of market information;
2) expands the market area due to increased buying and selling by description;
3) increases the level of competition for a product since the market

is larger; and

4) aids in transmission of price signals between consumers and producers, directing the highest quality of the product to the end use of greatest value.

Through these factors, grading more closely aligns the market for a particular commodity with that of perfect competition. Thus, the various grades of the product are channeled more efficiently to the various levels of demand.

Furthermore, a grading system must be operational and functional. In order to fulfill the economic purposes of grading, the factors to be graded must reflect those quality characteristics demanded by users. These factors must be easily, uniformly, and accurately measurable. The grading system should be simple and widely understood, with the same terminology used at all levels of the marketing process. Finally, the costs of operating the system must be reasonable.

Grades are usually, though not exclusively, established and administered by governments. Many agricultural products are graded, from fruits and vegetables to grains and oilseeds to livestock and meats. In particular, this study considers the grading of corn.

1.3. The Grading of Corn

Under the U.S. Grain Standards Act of 1916, which provided the first uniform national standards for the grading of grains, corn is divided into three classes, depending upon the shape, texture, and color of the kernel. The three classes of corn are Yellow Corn, White Corn, and Mixed

Corn. The classification of corn is further broken down into numerical grades, No. 1 through No. 5. These are based on the quality characteristics of moisture content, test weight, broken corn and foreign material (BCFM), heat damage, and total damage.

Moisture content. Moisture content is the amount of water in the kernel and indicates the amount of dry matter in the corn. Moisture content is important in two respects: 1) Buyers are purchasing the corn for the dry matter and nutrients that it contains, not for the water. Therefore, for most purposes, lower levels of moisture content are desired. 2) The amount of moisture is directly related to the storability of the corn--the higher the moisture content, the more likely is the deterioration and spoilage of the grain. Yet, corn kernels that are too dry are brittle and easily damaged, becoming more susceptible to contamination by mold, insects, or other infestations. For a particular sample of corn, moisture content is the percentage of the corn that consists of water, and is usually measured in the trade by electronic moisture meters. The reference standard method for moisture measurement is to place a preweighed sample of corn in an air oven at 103°C. for 72 hours, weighing it again upon removal, and calculating the percentage weight loss.

Test weight. Test weight is a volume measurement in pounds per bushel. It measures the density of the kernel and thus serves as an indicator of the amount of the grain that can be recovered through processing. Corn of high test weight generally contains more nutrients and less fiber, resulting in higher yield of processed products.

Broken corn and foreign material (BCFM). Broken corn consists of pieces of the corn kernel or cob fragments. Foreign material refers to extraneous material in the grain, such as dirt, weed seeds, and other grains. BCFM is any matter in the sample which will pass readily through a 12/64-inch seive, plus any nongrain material which can be hand-picked from the sample. BCFM is also an indicator of deterioration in that, as fine material, it packs closely, restricting air flow. BCFM is measured as a percentage of the total sample.

Total damage and heat damage. Damaged kernels provide less nutrient value and may cause losses in processing. Damage may be due to mold, frost, fungus, sprouting, disease, insects, or heat. Damage is also measured as a percentage of the total sample.

Table 1.1 gives the current U.S.D.A. grades and grade requirements for corn. The numerical grade classification is determined by the lowest quality grading factor. For example, a sample of corn may grade No. 2 for all factors except, say, moisture content, for which it falls into the No. 4 category. The entire lot of grain is then graded as No. 4 corn.

1.4. The Economics of Substandard Quality

U.S. No. 2 corn is accepted by the country grain trade as the standard contract grade. Corn exhibiting excessive levels of the properties listed in the official standards for No. 2 (see Table 1.1) is subject to market discounts. Since the quality characteristics of a lot of corn may be changed, either advertantly or inadvertantly, during its

Table 1.1. Grades and grade requirements for corn[a]

Grade	Minimum test weight per bushel (Pounds)	Maximum limits of...			
				Damaged kernels	
		Moisture (Percent)	Broken corn and foreign material (Percent)	Total (Percent)	Heat-damaged kernels (Percent)
U.S. No. 1	56.0	14.0	2.0	3.0	0.1
U.S. No. 2	54.0	15.5	3.0	5.0	0.2
U.S. No. 3	52.0	17.5	4.0	7.0	0.5
U.S. No. 4	49.0	20.0	5.0	10.0	1.0
U.S. No. 5	46.0	23.0	7.0	15.0	3.0
U.S. Sample Grade	U.S. sample grade shall be corn which does not meet the requirements for any of the grades from U.S. No. 1 to No. 5, inclusive, or which contains stones; or which is musty, or sour, or heating; or which has any commercially objectionable foreign odor; or which is otherwise of distinctly low quality.				

[a]From Official Grades and Standards of the United States (U.S.D.A., 1970).

journey through marketing channels, it may be tested, graded, and discounted at several levels of the marketing chain. This study focuses on the grading procedure at the level of first exchange, the movement of grain from the producer to the country elevator. Most of the corn marketed in Central Iowa and the Midwest moves via this route.

The procedure is basically as follows: A farmer delivers his grain to his local country elevator either for sale or for storage. At the elevator, a sample is taken from each load using a mechanical probe. The sample is then tested for levels of the various quality characteristics. If these levels are below the standards necessary for contract grade No. 2, market discounts will be applied according to the amount of deviation within each characteristic. The total amount of the discount will then be deducted from the price bid for the corn, reducing the farmer's revenue from the sale.

Moisture discounting occurs in one of two ways. If the grain is delivered to the elevator for storage, a fee is charged for drying. This fee is usually determined as cents per percentage point of moisture removed per wet bushel delivered. In addition, a weight shrink factor of 1.35 percent per percentage point of moisure removed is applied to the original delivery weight. Since corn is usually dried to 14.0 percent moisture content for storage purposes, the "percentage points removed" is the difference between the moisture content of the delivered wet grain and 14.0 percent. Thus, the drying charge depends on the moisture content determined by testing the sample at delivery.

If the grain is delivered to the elevator for immediate sale, a moisture discount is charged if the corn contains excess moisture. This discount covers the cost of drying the corn to acceptable trade levels, and is usually charged as a specified percent of the sale price. Again, it depends on the moisture content level determined by testing the sample at delivery.

It is important to note that the country elevator does not purchase grain from the farmer on the basis of the numerical grade. Instead, it discounts for individual properties. Thus, the value of the corn to the farmer is not based on the numerical grade, but on the size of the discounts per quality characteristic. And, the amount of the discounts depends on the difference between the properties as measured in the elevator's tests and the standards.

In economics, we are concerned with the value of a good, how that value is determined, and the effects of that valuation on the participants in the exchange of that good. Since discounting for substandard quality characteristics results in reduction of the value of corn to the producer, grading and discounting practices and procedures must be examined more closely.

When a load of corn comes into a country elevator, a sample is taken and tested for the various quality properties. The test results are then used to calculate the discounts, if any, to be charged. Since a single transaction may involve hundreds, perhaps thousands, of bushels of grain, an inaccuracy of only a couple of percentage points in the values obtained from the sample may result in a substantial monetary loss (or gain) to the producer.

During autumn harvest, when a large amount of corn moves directly from the field to the country elevator, the largest deviation from the standard U.S. Grade No. 2 occurs in the quality characteristic of moisture content. It is not difficult to see that moisture content represents the largest source of discounting of corn. Thus, errors in the determination of moisture levels by electronic moisture meter may have a strong effect on the valuation, or perhaps the misvaluation, of corn to the producer.

1.5. Sampling and Measurement Error

Two sources of error are readily apparent in the testing procedure for any quality characteristic: sampling error and measurement error. Sampling error may be due to problems with the sampling devices or methods used. After a sample has been taken from the load, it is measured for the various quality characteristics. Measurement error may result from the instruments used or from the techniques employed.

This study examines measurement error in determining moisture content in corn. Previous research procedures and results are useful and will now be reviewed.

Sampling and testing procedures have been evaluated with respect to determining BCFM content in corn (Hurburgh and others, 1979a). Several probing devices, those commonly used to obtain the sample from the load of grain, were tested and compared. Analysis showed that the in-load suction device, which pulls the sample into the collection tube by a vacuum, clearly collected excessive amounts of foreign material. Based on these results, the Iowa Legislature banned the use of the in-load suction device for obtaining samples to be tested for BCFM content.

Also investigated was the relationship between the location in the load from which the sample is taken and the accuracy of the test for BCFM. Samples taken from the center of the load were found to contain higher than the average amount of BCFM for the load, whereas samples from the extreme corners contain lower than the average amount of BCFM. Therefore, these locations should be avoided when collecting a sample to be measured for BCFM content. In addition, it was recommended that at least two samples be taken from the load and averaged. Even if center and corner locations are avoided, there may still be a difference of as much as one to five percentage points between the sample and the load average, if only one sample is taken (Hurburgh and others, 1979a).

In further research, moisture content sampling and measurement errors and BCFM sampling errors have been identified as being sufficient to cause financial losses. Research results suggested that the buyer, rather than the seller, benefits more often from these mistakes (Hurburgh and others, 1979b).

Using data collected by Hurburgh and his research team (1979a and 1979b), Udoh (1979) estimated the parameters of a probability distribution (assuming that the errors are normally distributed with constant variance) in order to determine the probability of improper discounting for moisture content and for BCFM content. For example, the probability that a sample of corn with a given true measure (considered to be the oven measure) would be discounted when it should not be, was calculated. This may also be thought of as the probability that the moisture meter will give an incorrect reading when the moisture content is, in fact, 15.5 percent, meaning that the corn should not be discounted. Udoh's results showed that corn with a true moisture content of 15.5 percent faces a probability of between eight and fifty-six percent of being discounted, due to errors in sampling and measurement. In addition, Udoh translated the measurement and sampling error amounts into dollar values meaningful to corn producers, concluding that discount error is a function of both types of error (Udoh, 1979).

Using corn samples collected during the 1979 harvest, Hurburgh and his co-workers (1980) turned their attention to measurement error in moisture content testing. In the laboratory, corn samples were tested for moisture content using electronic meters of the brands and types used by country elevators and by federal grain inspectors, and using the reference standard method of 72 hours in an air oven at 103°C. For each brand of meter, measurement error, the difference between the meter reading and the oven measure, was plotted against moisture content determined by the oven method, and a "calibration correction line," or curve, was fit to the data, using ordinary least squares regression methods. For all meters except one, the estimated deviations from the oven measure increased through the mid-range of moisture content, then decreased.

The calibration correction equation was significant for all meters studied, including the Motomco meter used by the Federal Grain Inspection

Service, which is calibrated to the oven measure. The Steinlite meter, the brand used by over 75 percent of the country elevators in Iowa, was one of two meters to show the largest error (Hurburgh and others, 1980). These results were used in a massive effort to recalibrate the moisture meters in Iowa before the 1980 harvest, with focus placed on moisture content in the low to middle ranges.

Meter precision was also considered in this study. It was concluded that, as moisture content increased, the precision of the meters decreased, and, thus, their variability increased (Hurburgh and others, 1980).

Research in this area continued, with similar analysis carried out on samples of corn collected during the 1980 harvest. Again, calibration correction equations were estimated and used to modify previous recalibration of the meters, this time with focus placed on the higher ranges of moisture content (Hurburgh and others, 1981). Recalibration was completed before the 1981 harvest.

This study uses Hurburgh's data from 1979, before recalibration began, and data from the 1981 and 1982 harvests, after recalibration was complete, in an additional attempt to analyze moisture content measurement errors, as well as to evaluate the effectiveness of meter recalibration. The goals are similar to those of Udoh--to determine the probability of error in moisture content measurement. The results will set the stage for further research into the economics of more reliable determination of moisture content in the grading of corn.

2. OBJECTIVES

The objective of this study is to evaluate the effects of recalibration of the moisture meters on measurement error in determining moisture content in corn. Since data from before and after recalibration are available, we have the opportunity to analyze the data from both periods and draw a comparison. In other words, we are attempting to answer the question: Did recalibrating the meters significantly affect the accuracy of measuring corn for moisture content at country elevators?

More specifically, for each period of data, the probability that a load of corn will be misgraded with respect to moisture content will be determined, based on the true moisture content. The major focus of this study will be on the discount decision--that is, the decision to discount or to not discount the corn for excessive moisture content. Although measurement error affects the size of the discount and/or drying charges as well, that problem will receive only minor attention here. Finally, the results of the analysis of each data set will be compared so that a judgment may be made as to whether this probability has changed since recalibration of the meters.

This is basically an extension of the work done by Udoh, with four major differences:

1) The data base is larger and broader, with corn samples being obtained from various locations throughout the country for several years.

2) A change has occurred since Udoh's study--the recalibration of the moisture meters. Since data is available from both before and after this change, its effects may be evaluated. Thus, this study involves a comparison.

3) Udoh's research dealt with both sampling and measurement errors and with combinations of the two. Here we will focus exclusively on measurement error in determining moisture content.

4) Udoh assumed that the error variance was constant over the entire range of moisture content. Evidence from Hurburgh's studies suggests that this is not true. Instead it appears that the variance of the measurement errors, and, thus, of the meter measure, widens as moisture content increases. In this study, the existence of nonconstant error variances, if supported by appropriate statistical evidence, will be incorporated into the research procedure.

3. THE DATA

Recalibration of the electronic moisture meters took place during the period 1980-1981, resulting from extensive research undertaken by the Illinois-Iowa Moisture Meter Task Force. To facilitate their study, data sets were generated from samples of corn collected during the harvests of 1979, 1980, 1981, and 1982, before, during, and after meter recalibration. Portions of these data sets make up the data base for this study.

The two periods of data sets relevant to this study are those from 1979, before recalibration took place, and 1981 and 1982, after recalibration was complete. The data from 1981 and 1982 will be pooled into one data set. Because meter calibration was the same for both these years, the 1981 and 1982 data sets can be assumed to be representative of the same population.

Samples of corn from various locations around the country were provided to the Grain Quality Research Laboratory of the Department of Agricultural Engineering at Iowa State University. Distribution of these samples by place of origin for 1979, 1981, and 1982 is given in Table 3.1.

Table 3.1. Distribution of corn samples by place of origin

Origin	1979	1981	1982
Iowa	196	806	977
Illinois	15	0	0
Outside Iowa and Illinois	101	195	72
Total	312	1001	1049

Elaborate laboratory procedures were designed for dividing each corn sample into subsamples which were tested for various quality characteristics, including moisture content. Details about the experimental design may be found in Hurburgh's dissertation (Hurburgh, 1981a). All testing was done in the laboratory. Two methods were utilized to test for the moisture content of the corn.

The reference standard method for determining moisture content was the air-oven method, as specified in Chapter XXI, Grain Equipment Manual GR 916-6, Federal Grain Inspection Service. This procedure involves placing a pre-weighed sample of corn in an air-oven at 103°C for 72 hours, and weighing the sample again upon removal. The percentage weight-loss is the percentage moisture content.

Hurburgh's studies show that the variability of the oven method is not related to moisture content. With regard to the experimental design of his studies, the precision of the oven method fell in the realm of the accepted precision stated in the government standards (Hurburgh and others, 1980, p. 12 and Hurburgh, 1981a). Furthermore, according to Hurburgh, "The internal variance of the oven determination was small compared to the variance of a meter-to-oven comparison. Improvements in the precision of the reference method will not contribute significantly to more accurate meter calibrations" (Hurburgh, 1981a, pp. 108-109). Thus, the variability of the oven method may be considered to be a random component of the variability of the meter measure. For these reasons, this study will consider the oven method to provide the true value of moisture content of the corn. The variable T will denote true moisture content.

The second method for measuring moisture content was by electronic moisture meters. The meters used were provided to the Grain Quality Research Laboratory by the manufacturers, and were representative of meters sold for commercial use (Hurburgh, 1980, pp. 5-6). The meters considered in this study are the Burrows 700, the Dickey-john GAC II, the Motomco 919, and the Steinlite SS250. The Steinlite SS250 is the most common meter to be found at Iowa's country elevators. The Motomco 919 is the meter used by the Federal Grain Inspection Service to measure moisture content in all corn that is sold in international commerce, and on all internal trades where U.S. grades are requested.

Subsamples of corn were tested for moisture content using both methods. A portion of the subsample was tested using the oven method, which gave the value of its true moisture content, T. Another portion of the subsample was tested for moisture content with an electronic moisture meter. Three meter tests were performed on each subsample, giving three meter readings. The mean of these three readings was reported for each observation. Let M be the variable name denoting the individual meter readings, and $\overline{M}$ be the variable name denoting the mean of three meter readings on the same subsample. In addition, the sample variance of the three original meter readings was computed and is denoted by the variable name V^s.

Thus, each observation for each year for each meter contains values of the following variables:

T = true moisture content.

$\overline{M}$ = the mean of three individual meter readings of moisture content, the M_j, where $j = 1, 2, 3$.

V^s = the sample variance of the three individual meter readings.

Let subscripted small letters denote the observational values of these variables. That is, t_i is the value of true moisture content for the i-th subsample of corn; the m_{ij}, $j = 1, 2, 3$, are the individual meter readings on the i-th subsample; and so on. Then, the i-th reported observation is obtained from the steps diagramed in Figure 3.1. There are n reported observations on $\overline{M}$ and V^s in the data sets used in this study. Thus, there would have been 3n original observations on M, the individual meter readings.

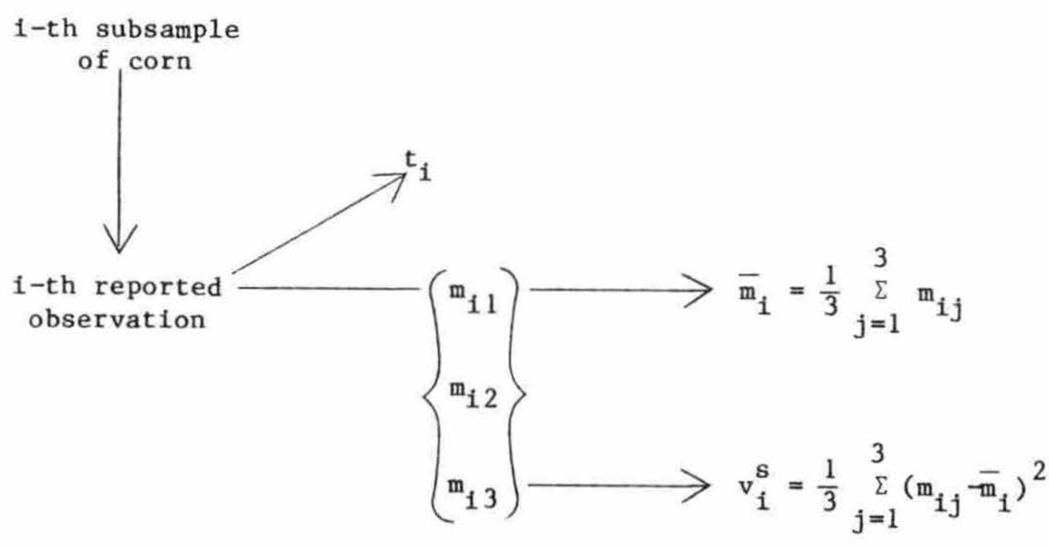


Figure 3.1. Steps by which each reported observation is obtained

A summary of simple descriptive statistics for each variable by meter brand and by period is displayed in Table 3.2. It must be noted that there are no 1982 data for the Burrows 700 moisture meter, since the manufacturer has discontinued its production. Thus, the 1981-82 period data for the Burrows actually contains observations for 1981 only.

Table 3.2. Descriptive statistics for each variable by meter and period

Meter brand	Period	Number of observations n	Moisture content						Variance, V		
			Mean		Minimum		Maximum		Mean	Minimum	Maximum
			$\overline{M}$	T	$\overline{M}$	T	$\overline{M}$	T			
Burrows	79	611	20.64	19.94	11.47	9.31	39.67	37.27	0.101	0.00	22.468
	81-82	1,000	22.63	22.80	9.67	6.17	41.43	40.82	0.080	0.00	1.613
GAC II	79	602	20.13	19.95	10.90	9.27	37.90	37.46	0.055	0.00	1.538
	81-82	2,055	23.07	22.80	6.93	6.17	42.73	40.82	0.082	0.00	3.209
Motomco	79	606	19.73	20.08	10.22	9.31	34.62	37.25	0.100	0.00	19.536
	81-82	2,055	22.58	22.80	6.57	6.17	50.60	40.82	0.096	0.00	2.595
Steinlite	79	596	20.50	19.82	11.27	9.27	35.57	37.46	0.046	0.00	3.423
	81-82	2,055	22.68	22.80	6.71	6.17	40.93	40.82	0.056	0.00	2.392

4. ANALYTICAL RESEARCH PROCEDURE

4.1. Introduction

This chapter describes the research procedure used to analyze the data from each period, before and after meter recalibration. The results of these analyses will be compared so that the extent of the effects of recalibration can be evaluated.

For the data from each period, the final result of the analysis will be the probability of the decision to discount being made incorrectly for a given level of moisture content. An incorrect discount decision will be the result of error in measuring the moisture content of the corn--that is, if the meter measurement differs from the true value.

Let M = meter measure, T = true moisture content, and E = measurement error, all in percent. Then, $E = M - T$, or $M = T + E$. Previous studies have demonstrated that measurement error is related to the moisture content of the corn. Thus, measurement error may be assumed to be a function of true moisture content: $E = f(T)$. Since E is a function of T, M must also be a function of T: $M = T + f(T) = g(T)$. This study examines the problem of measurement error through the assumption of $M = g(T)$.

We are ultimately interested in determining the probability of measurement error. The question being asked, then, in terms of M and T, is: Given a particular value of T, what is the probability that M will differ from T? In order to calculate probabilities, one needs a distribution. Here we will assume that, for a given level of moisture content, meter measures are drawn from a normal distribution. In order to specify the distribution, one needs its mean and variance. These may be estimated from the data.

Let M_T be the meter measurement associated with true moisture content, T. Since meter values are drawn from a normal distribution, $M_T \sim N(\mu_T, \sigma_T^2)$. M_T, as a general term, stands for a family of random variables, each distributed normally. μ_T is the family of means of these random variables; σ_T^2 is the family of variances. The members of the families M_T, μ_T, and σ_T^2 are each conditional upon the level of true moisture content, T. They are specified when a particular value of T is specified. Given that $T = \tau$, where τ is some numerical value of T, M_τ is the random variable whose distribution is the distribution of meter measurements associated with true moisture content, $T = \tau$. M_τ is a particular member of the family of random variables, M_T. $M_\tau \sim N(\mu_\tau, \sigma_\tau^2)$, where μ_τ and σ_τ^2 are members of the families μ_T and σ_T^2, respectively. In summary, M_τ is a particular random variable with a particular distribution, namely $N(\mu_\tau, \sigma_\tau^2)$.

In order to specify the distribution completely, exact numerical values of the mean and the variance are required. Therefore, we need estimates of the distributional parameters, μ_τ and σ_τ^2.

One method of estimating these parameters is simply to calculate the sample mean and the sample variance from the raw data. At each value of T, there is a distribution of meter measurements. The raw data provides a sample from this distribution at each T. For a given value of true moisture content, say $T = \tau$, $\overline{M}_\tau$, the sample mean of meter measurements, and s_τ^2, the sample variance of meter measurements may be calculated. These values may then be used as the parameter estimates in specifying the probability distribution of M_τ. Note that $\overline{M}_\tau$ and s_τ^2 are computed from only the scatter of points at $T = \tau$ (see Figure 4.1) and, thus, are totally unrelated to the sample mean and the sample variance at any other value of T. In other words, the scatter of meter measurements at each value of T is considered a single sample, independent of every other value of T.

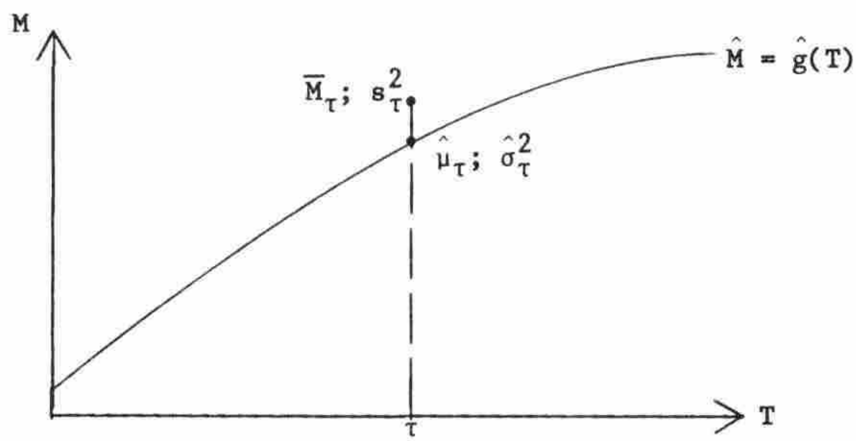


Figure 4.1. Graphical illustration of two methods for estimating the distributional parameters

However, as noted earlier, there is a systematic relationship between the meter measurements and true moisture content across the entire range of moisture content. M may reasonably be assumed to be a function of T. In particular, μ_T, the mean of the distribution about M_T, is assumed to be a function of T: $\mu_T = g(T)$. This function can be estimated from sample data using statistical regression analysis. The estimated equation would be $\hat{\mu}_T = \hat{g}(T)$. Knowledge of these results can be used in calculating estimates of the distributional mean and variance, $\hat{\mu}_\tau$ and $\hat{\sigma}^2_\tau$, at each T = τ (see Figure 4.1). This procedure recognizes an underlying relationship between meter measurement and true moisture content, and, thus, utilizes more information than the first method described above. There, estimates of the individual means and variances use only the information at the particular individual level of moisture content being considered, whereas our estimates of the individual means and variances will draw upon information from the entire range of moisture content.

The purpose of this chapter is twofold. First, regression theory will be applied in the estimation of a variance function, the results of which will be incorporated into the estimation of $M = g(T)$, the relationship between the meter measure of moisture content and the true value. We will show how the usual techniques are varied in order to accomodate the complications due to the nature of the problem and/or the data. Second, we will describe how the regression results will be applied in calculating the individual means and variances, and the desired probabilities.

4.2. Statistical Regression Analysis

4.2.1. Heteroscedasticity and the variance function

The linear model describing the relationship between individual meter measurements of moisture content and the true moisture content, in matrix form, is

$$M = T\beta + U, \qquad (4.1)$$

where M is a 3n x 1 column vector of original meter readings, T is 3n x k, β is k x 1, and U is a 3n x 1 column vector of random errors. In addition, k is the number of independent variables for which coefficients will be estimated, 3n is the number of individual meter readings, and n is the number of reported observations. M, T, and U are specified in detail in section 4.2.2 and the Appendix. Under the theory of ordinary least squares (OLS) regression, the assumption of homoscedasticity says that the variance of the stochastic error term is constant over the entire range of the independent variable. Furthermore, the variance of the dependent variable is equal to the variance of the error term. Here, if OLS were applied, we would be assuming that $\text{var}(U) = \sigma^2 = \text{var}(M)$ is homoscedastic. That is, under OLS, the variance of the meter measure, M, would be assumed to be constant over the entire range of true moisture content, T. Previous studies have suggested that this is not necessarily true. It is believed that the variance of measurement error, $E = M - T$, is nonconstant relative to T, so that the variance of the meter measure is nonconstant, as well.

Violation of the assumption of equal variances, the existence of unequal variances, is known as heteroscedasticity in the language of econometrics. Heteroscedasticity is not uncommon in studies such as this, which are based on cross-sectional data. Since previous research suggests that the variance of the meter measurement is nonconstant relative to the level of the true moisture content, it is necessary that this hypothesis be tested statistically.

Several tests for heteroscedasticity are available. Some are general tests simply to answer the question of its existence; others try to identify the nature of the heteroscedasticity and usually presuppose some functional form of the individual variances in terms of the independent variable. Only the test chosen for application in this study will be discussed here. For information about other tests, the interested reader is referred to the econometrics text by Judge, Griffiths, Hill, and Lee, which contains an entire chapter on heteroscedasticity, the various tests for its existence, and the situations in which each is most applicable (Judge and others, 1980).

The test for heteroscedasticity applied in this study is based on a variation of the Park-Glejser test (Pindyck and Rubinfeld, 1981). This test is desirable because it will allow us to test for the existence of heteroscedasticity, as well as its form. The variation of interest to us involves regressing the absolute value of OLS estimated residuals on the independent variable, and testing the significance of the parameter estimates. Since most economic data consist of one replication per observation on the independent variable, the OLS estimated residuals may be thought of as "sample variances" at each point. But, as will be recalled from Chapter 3 describing the data, each of our observations includes a sample variance of three meter readings per true value. Thus, we are able to take advantage of the available data in application of this test for heteroscedasticity.

Let V be the true variance of three individual meter readings. Since the variance of the meter measurements is believed to be related to the true moisture content, we assume that V is a function of T: $V = h(T)$. This relationship can be estimated using the sample variances of three meter readings at each value of true moisture content. Let V^s denote this n x 1 column vector of sample variances from the reported data. A typical element of V^s is

$$v_i^s = \frac{1}{3} \sum_{j=1}^{3} (m_{ij} - \bar{m}_i)^2,$$

where

$$\bar{m}_i = \frac{1}{3} \sum_{j=1}^{3} m_{ij}$$

and the m_{ij} are the individual meter readings, $i = 1, 2, \ldots, n$, $j = 1, 2, 3$, and n is the number of reported observations.

The model to be estimated, in matrix form, is

$$V^s = \bar{T}\alpha + W, \tag{4.2}$$

where α is a k x 1 vector of parameters, $\bar{T}$ is n x k and is specified in the Appendix, and W is an n x 1 column vector of stochastic homoscedastic error terms. The ordinary least squares assumptions are:

1) $E(W) = 0$.
2) $E(WW') = \sigma^2 I$, or, equivalently,
   a) $E(w_i^2) = \sigma^2$ for all i.
   b) $E(w_i w_j) = 0$ for all $i \neq j$.
3) $E(\bar{T}'W) = 0$.

4) The independent variables (the columns of $\overline{T}$) are linearly independent.

5) The errors are normally distributed: $W \sim N(0, \sigma^2)$.

Under OLS, the best linear unbiased estimate of α is

$$\hat{\alpha} = (\overline{T}'\overline{T})^{-1} (\overline{T}'V^s),$$

and the variance-covariance, or dispersion, matrix of $\hat{\alpha}$ is

$$\text{var}(\hat{\alpha}) = D(\hat{\alpha}) = (\overline{T}'\overline{T})^{-1}.$$

Two functional forms of the model in equation (4.2) will be estimated: linear, with $k = 2$, and quadratic, with $k = 3$. The goodness of fit statistics from the estimation will be used to judge which form is more appropriate. If the slope coefficient is significantly different from zero, then the assumption of heteroscedasticity will be adopted and incorporated into the succeeding analysis.

The resulting estimated equation will be referred to as the estimated variance fucntion. Letting T'_τ be an arbitrary 1 x k row vector for a given value of the moisture content, $T = \tau$, the estimated variance function is

$$\hat{v}^s_\tau = T'_\tau\hat{\alpha},$$

where $\hat{v}^s_\tau$ is the estimated variance at $T = \tau$. In particular, when $k = 3$, $T_\tau = (1, \tau, \tau^2)$ and the estimated variance function is

$$\hat{v}^s_\tau = \hat{\alpha}_1 + \hat{\alpha}_2\tau + \hat{\alpha}_3\tau^2.$$

Finally, $\hat{V}^s$ is an n x 1 column vector of estimated variances calculated for each observation on $\overline{T}$:

$$\hat{V}^s = \overline{T}\hat{\alpha}.$$

An estimate of the variance function is valuable to this study in two respects. It provides a specification of the form of the heteroscedasticity to be used in estimating the model M = g(T) + U, based on the assumption var(M) = var(U) = V, where V is a diagonal matrix whose main diagonal elements are the true variances of three individual meter readings. Secondly, it will be used in estimating the variance of the distribution of M at each T.

4.2.2. Generalized least squares estimation of M = g(T)

Heteroscedasticity violates the classical assumptions of ordinary least squares, making OLS inappropriate for estimating the relationship between meter moisture content and true moisture content. The model and assumptions of generalized least squares (GLS) estimation do accommodate the existence of heteroscedasticity. Thus, if heteroscedasticity is supported by the significance of the estimated variance function in section 4.2.1, then GLS will be applied.

The linear model describing the relationship between individual meter measurement of moisture content and true moisture content is

$$M = T\beta + U \tag{4.3}$$

with the following GLS assumptions:

1) $E(U) = 0$.

2) $E(UU') = V$.

3) $E(T'U) = 0$.

4) The independent variables (the columns of T) are linearly independent.

5) The errors are normally distributed.

M, T, β, U, k, 3n, and n are as defined following equation (4.1) in section 4.2.1. M, T, and U are specified in the Appendix. In particular, notice assumption (2). This is the assumption of heteroscedasticity. V is a 3n x 3n diagonal matrix of known values, by assumption. Specifically,

$$V = \begin{pmatrix} v_1 & & & & & & & & & & & \\ & v_1 & & & & & & & & 0 & & \\ & & v_1 & & & & & & & & & \\ & & & v_2 & & & & & & & & \\ & & & & v_2 & & & & & & & \\ & & & & & v_2 & & & & & & \\ & & & & & & \cdot & & & & & \\ & & & & & & & \cdot & & & & \\ & & & & & & & & \cdot & & & \\ & & 0 & & & & & & & v_n & & \\ & & & & & & & & & & v_n & \\ & & & & & & & & & & & v_n \end{pmatrix}$$

where each v_i is the true variance of three individual meter readings on the same sample. The v_i are also assumed to be a function of true moisture content, such that the variance grows as true moisture content increases. In addition, the V-matrix, in order to satisfy GLS assumptions, must be nonsingular (invertible) and positive definite. (The definition of positive definite may be found in Johnston, 1972, pp. 105-106.)

Applying GLS estimation techniques, the best linear unbiased estimate of β in equation (4.3) is

$$\hat{b} = (T'V^{-1}T)^{-1} (T'V^{-1}M). \tag{4.4}$$

The variance-covariance, or dispersion, matrix of $\hat{b}$ is

$$\text{var}(\hat{b}) = D(\hat{b}) = (T'V^{-1}T)^{-1}. \tag{4.5}$$

The model in equation (4.3) and the estimation results in equations (4.4) and (4.5) are for the original data, which include the individual meter readings of moisture content. But these original data were not reported. In fact, the reported data, as noted in Chapter 3, consist of the means of readings from three drops of a corn sample through the moisture meter. This grouping of the original observations removes some of the variability that we are attempting to study, and brings an additional complication to the estimation procedure.

For the theory of regression applied to grouped observations, the reader is advised to consult Johnston, Econometric Methods, chapter 7 (Johnston, 1972). There the procedure is described specifically for the case of an OLS problem, where grouping of data results in a heteroscedastic error term, thus requiring GLS techniques instead.

In this study, the original data are believed to be heteroscedastic and are described by a GLS model. Thus, our final estimation procedure must accommodate both heteroscedasticity in the individual meter readings and the grouping of these observations.

The relationship between the group means in our reported data and the original observations is

$$\overline{M} = GM,$$

$$\overline{T} = GT,$$

and

$$\overline{U} = GU,$$

where G is an n x 3n grouping matrix. Since each grouped observation is the mean of three original observations,

$$G = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \dots & & & & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \dots & & & \\ & & & & \ddots & & & \ddots & & & & & \\ 0 & 0 & 0 & & & \dots & & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

$\overline{M}$ and $\overline{U}$ are n x 1 column vectors of group means; $\overline{T}$ is n x k.

The model in terms of grouped data is

$$\overline{M} = \overline{T}\beta + \overline{U} \tag{4.6}$$

where the error variance structure is

$$\text{var}(\overline{U}) = \overline{E}(\overline{UU'}) = E(GUU'G')$$

$$= G(E(UU'))G' = GVG'.$$

The GLS estimate of β is

$$\hat{b}^G = [\bar{T}'(GVG')^{-1}\bar{T}]^{-1} [\bar{T}'(GVG')^{-1}\bar{M}]. \tag{4.7}$$

The notation $\hat{b}^G$ specifies that this GLS estimate was calculated from grouped data. The variance-covariance matrix, or dispersion matrix, of $\hat{b}^G$ is

$$var(\hat{b}^G) = D(\hat{b}^G) = [\bar{T}'(GVG')^{-1}\bar{T}]^{-1}. \tag{4.8}$$

It can be shown that $\hat{b}^G = \hat{b}$ and $var(\hat{b}^G) = D(\hat{b}^G) = D(\hat{b}) = var(\hat{b})$. Some of the other regression results differ, but the relationship between the two can be derived algebraically. The derivation of these and other results concerning the application of the theory of grouped observations to this problem can be found in the Appendix. Nevertheless, the grouped data can be used to compute $\hat{b}$, the estimate of the parameters of the model describing the original observations.

In order to actually carry out the estimation of $\hat{b}^G$, we must consider the V-matrix further. Since the elements of the V-matrix are the true variances of three meter readings, they are not known values as required by GLS. But they can and will be estimated by the techniques described in section 4.2.1. Let $\hat{V}$ be the estimate of V. The main diagonal elements of the $\hat{V}$-matrix, the $\hat{v}_i$, will be the elements of the n x 1 column vector of estimated variances, $\hat{V}^s$, where

$$\hat{V}^s = \bar{T}\hat{\alpha},$$

as defined in section 4.2.1. The estimation of $\hat{b}^G$ and $D(\hat{b}^G)$ will proceed with $\hat{V}$ replacing V in equations (4.7) and (4.8).

Finally, we must consider briefly the question of functional specification of the model in equation (4.6). Is the relationship between meter measurement and true moisture content linear or nonlinear? Plots of the data and results of previous studies suggest that the relationship between measurement error and true moisture content is quadratic. If this is so, then the functional form of M with respect to T would also be quadratic. To test this statistically, both linear ($k = 2$) and quadratic ($k = 3$) forms of equation (4.6) will be estimated. Appropriate goodness of fit statistics will be considered to judge which of the two forms is better. The analysis will then proceed using the selected model.

4.3. Testing for the Equality of the Parameters

The purpose of this study is to evaluate the effects of recalibration of the moisture meters between 1979 and 1981. Thus, we wish to compare the results of the data analysis from each period for each meter. At this point, these results consist of estimates of the relationships between the meter measure and true moisture content. The regression results from the two periods can be compared by testing the parameters from each period for equality.

Section 4.2.2 modelled $M = g(T)$ with equation (4.3) for the original data and equation (4.6) for the grouped data, and showed that the β-parameters in these models can be estimated by $\hat{b}^G$ given in equation (4.7), with $\hat{V}$ replacing V. The model will be estimated for each period, and the parameter estimates will be compared by using an F-test for the equality of parameters.

Let the subscript h, h = 1, 2, reference the data and results as to their period, with period 1 being 1979, before recalibration, and period 2 being 1981 and 1982 taken together, after recalibration. That is, β_1, $\hat{b}_1^G$, and n_1 are the parameters, the parameter estimates, and the number of grouped observations, respectively, for 1979, which is period 1. β_2, $\hat{b}_2^G$, and n_2 are similarly defined for the pooled data from 1981 and 1982, which make up period 2.

To test the null hypothesis $H_0 : \beta_1 = \beta_2$ against the alternative $H_A : \beta_1 \neq \beta_2$, we calculate the F-statistic:

$$F = \frac{\text{Num}}{\text{Denom}} \tag{4.9}$$

where

$$\text{Num} = (\hat{b}_1^G - \hat{b}_2^G)' \left\{ [\overline{T}_1'(G_1\hat{V}_1G_1')^{-1}\overline{T}_1]^{-1} + [\overline{T}_2'(G_2\hat{V}_2G_2')^{-1}\overline{T}_2]^{-1} \right\}^{-1} (\hat{b}_1^G - \hat{b}_2^G)/k$$

and

$$\text{Denom} = \frac{\text{ESS}(\hat{b}_1^G) + \text{ESS}(\hat{b}_2^G)}{3(n_1+n_2) - 2k} + 1.$$

$\text{ESS}(\hat{b}_h^G)$ denotes the error sum of squares, or residual sum of squares, associated with the estimation for period h, h = 1, 2, and k equals the number of parameters. The F-statistic of equation (4.9) possesses an F distribution with k and $3(n_1+n_2) - 2k$ degrees of freedom. If the observed F value is significant at the 5 percent level, then the null hypothesis will be rejected, supporting the alternative hypothesis that the parameters from the two periods are indeed different.

Details of the derivation of the above F-statistic are presented in the Appendix.

4.4. The Probability Distributions of Meter Measurement

4.4.1. Estimating the individual means and variances

Once again recall that we wish to determine the probability of error in measuring the moisture content in corn. These probabilities will be developed in terms of the meter measure, M, and the true measure, T. In general, $M_T \sim N(\mu_T, \sigma_T^2)$ and, in particular, $M_\tau \sim N(\mu_\tau, \sigma_\tau^2)$ for a given value of $T = \tau$. Estimates of these individual means and variances are necessary. The theory introduced in section 4.1 and the regression results from section 4.2 will be used in order to calculate estimates of μ_τ and σ_τ^2, the means and variances of the individual distributions of M_τ at each value of the moisture content, $T = \tau$.

Recall that the model for the original data, if they were available, is

$$M = T\beta + U$$

with the heteroscedasticity assumption

$$\mathrm{var}(U) = E(UU') = V. \tag{4.10}$$

The parameter estimates would be provided by

$$\hat{b} = (T'\hat{V}^{-1}T)^{-1}\,(T'\hat{V}^{-1}M),$$

where $\hat{V}$, the estimated variance matrix, replaces V in the actual estimation. The dispersion matrix for $\hat{b}$ is

$$D(\hat{b}) = var(\hat{b}) = (T'\hat{V}^{-1}T)^{-1}. \tag{4.11}$$

We wish to derive estimates of μ_T and σ_T^2 from these results. Call these estimates $\hat{\mu}_T$ and $\hat{\sigma}_T^2$, in general, and $\hat{\mu}_\tau$ and $\hat{\sigma}_\tau^2$ for $T = \tau$ in particular.

Let $T = \tau$ be represented in row vector form by T'_τ. For example, if the estimated model was quadratic, $T'_\tau = (1, \tau, \tau^2)$. Then an estimated meter reading is given by

$$\hat{M}_\tau = T'_\tau \hat{b}.$$

From regression theory, the value $\hat{M}_\tau$ provides an estimate of the conditional mean $\mu_{M|T=\tau} = \mu_\tau$ of the distribution of M at $T = \tau$. That is,

$$\hat{\mu}_\tau = \hat{M}_\tau = T'_\tau \hat{b} \tag{4.12}$$

is an estimate of μ_τ.

Derivation of the variance of the distribution of M_τ is a little more complicated. Since we will be calculating the probability with respect to a single meter reading drawn from the distribution of meter readings at $T = \tau$, the variance required here is that of a predicted value. In addition, it must include the effects of heteroscedasticity.

Note that, now, $\hat{M}_\tau$ takes on the alternative interpretation provided by regression theory, that of a single predicted value. The variance we need, then, is the variance associated with the predicted value, $\hat{M}_\tau$. Thus, given that $M_\tau = T'_\tau \beta + u_\tau$ is the actual meter reading at $T = \tau$, the variance of the predicted value as compared with the actual value is:

$$\begin{aligned} \mathrm{var}(\hat{M}_\tau) &= E[(\hat{M}_\tau - M_\tau)\ (\hat{M}_\tau - M_\tau)'] \\ &= E[(T'_\tau(\hat{b}-\beta)-u_\tau)\ (T'_\tau(\hat{b}-\beta)-u_\tau)'] \\ &= T'_\tau\{E[(\hat{b}-\beta)(\hat{b}-\beta)']\}T_\tau + E(u_\tau^2). \end{aligned} \tag{4.13}$$

Since $E[(\hat{b}-\beta)(\hat{b}-\beta)'] = \mathrm{var}(\hat{b})$, the first term of equation (4.13) above is equal to $T'_\tau[\mathrm{var}(\hat{b})]T_\tau$, and equation (4.11) is applicable. This is interpreted as the amount of variation in prediction due to parameter estimation. The second term of equation (4.13) is equal to v_τ, where v_τ is a diagonal element of the V-matrix in the heteroscedasticity assumption of equation (4.10). This is the amount of variation due to random error, which is heteroscedastic. Continuing,

$$\begin{aligned} \mathrm{var}(\hat{M}_\tau) &= T'_\tau[\mathrm{var}(\hat{b})]T_\tau + v_\tau \\ &= T'_\tau D(\hat{b})T_\tau + v_\tau \\ &= T'_\tau(T'\hat{V}^{-1}T)^{-1}T_\tau + v_\tau. \end{aligned}$$

An estimate of v_τ is

$$\hat{v}_\tau = T'_\tau\ \hat{\alpha}.$$

from the variance function estimated in section 4.2.1. Therefore, an estimate of σ_τ^2, the variance of the distribution of M_τ, is provided by

$$\hat{\sigma}_\tau^2 = \mathrm{var}(\hat{M}_\tau) = T'_\tau(T'\hat{V}^{-1}T)^{-1}T_\tau + \hat{v}_\tau. \tag{4.14}$$

Finally, recall from section 4.2.2 and Appendix A that

$$\hat{b} = \hat{b}^G = (\overline{T}'(G\hat{V}G')^{-1}\overline{T})^{-1}\ (T'(G\hat{V}G')^{-1}M)$$

and
$$D(\hat{b}) = D(\hat{b}^G) = (\overline{T}'(G\hat{V}G')^{-1}T)^{-1},$$

where $\hat{b}^G$ is the vector of parameter estimates for the model in equation (4.6) in section 4.4.2, which describes the grouped observations. Thus, the results in equations (4.12) and (4.14) concerning $\hat{\mu}_\tau$ and $\hat{\sigma}^2_\tau$ hold with $\hat{b}^G$ replacing $\hat{b}$ and $D(\hat{b}^G)$ replacing $D(\hat{b})$.

In summary, given a particular value of true moisture content, $T = \tau$, represented in row vector form by T'_τ, the estimates of the mean and the variance of the individual probability distribution of M_τ are:

$$\hat{\mu}_\tau = \hat{M}_\tau = T'_\tau\hat{b}^G \qquad (4.15)$$

and
$$\hat{\sigma}^2_\tau = \text{var}(\hat{M}_\tau) = T'_\tau D(\hat{b}^G)T_\tau + \hat{v}_\tau$$

$$= T'_\tau(\overline{T}(G\hat{V}G')^{-1}\overline{T})^{-1}T_\tau + \hat{v}_\tau. \qquad (4.16)$$

For illustration, see Figure 4.2.

4.4.2. Calculating the probabilities

Now that we have parameter estimates to use in specifying the probability distribution of each member of the family of random variables, M_T, we can proceed with the calculation of the conditional probability of error in measurement for a given level of true moisture content.

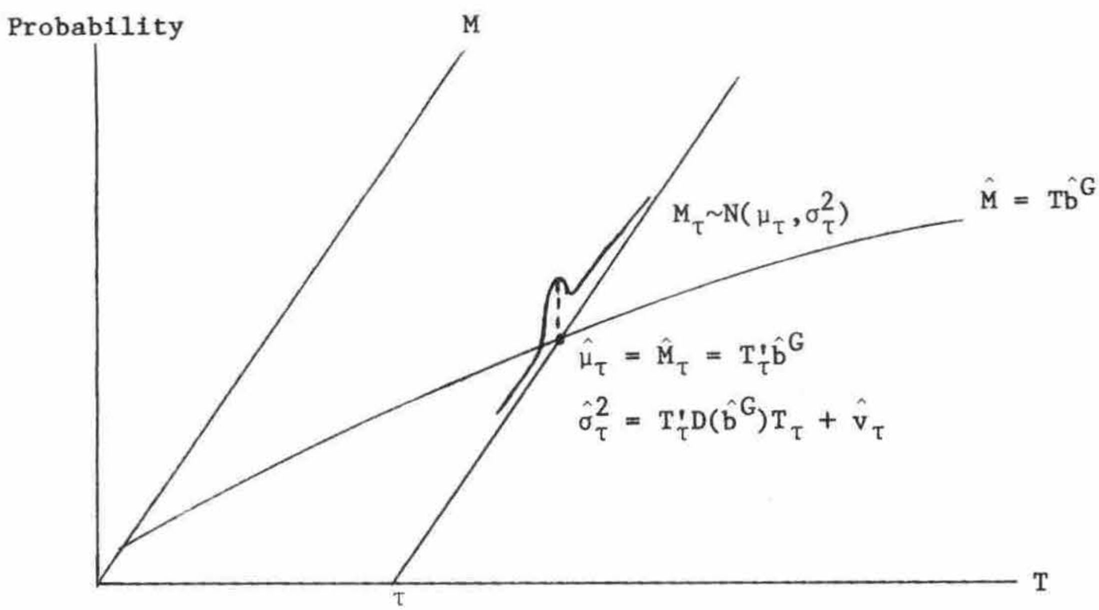


Figure 4.2. Graphical illustration of distributional parameter estimates obtained from regression results

Recall that, for a given $T = \tau$, $M_\tau \sim N(\mu_\tau, \sigma_\tau^2)$, where the estimates of μ_τ and σ_τ^2 are given by $\hat{\mu}_\tau$ in equation (4.15) and $\hat{\sigma}_\tau^2$ in equation (4.16), respectively. We wish to determine the probability that the meter measurement, M, will differ from a particular value of T. For example, given that $T = \tau$, we may be interested in the probability that the corresponding meter measurement, M_τ, will be greater than a specified value of M, say m. This may be computed as follows:

$$\Pr(M_\tau > m \mid T = \tau) = \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} > \frac{m - \hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z > z_{m,\tau}), \qquad (4.17)$$

where M_τ, the meter measurement associated with $T = \tau$, is a random variable with mean μ_τ and standard deviation σ_τ. Since the meter measurements are, by assumption, normally distributed, $\frac{M_\tau - \mu_\tau}{\sigma_\tau}$ has the standard normal distribution with mean equal to zero and standard deviation equal to one. That is, $\frac{M_\tau - \mu_\tau}{\sigma_\tau} = Z$, where $Z \sim N(0, 1)$.

In practice, μ_τ and σ_τ will be replaced by their estimates, $\hat{\mu}_\tau$ and $\hat{\sigma}_\tau$, respectively, calculated from equations (4.15) and (4.16), found in section 4.4.1. Because the sample from which these estimates are made is sufficiently large, the distribution of $\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau}$ so closely approximates the standard normal that it may be used in determining the probability in equation (4.17).

Finally, $\frac{m - \hat{\mu}_\tau}{\hat{\sigma}_\tau} = z_{m,\tau}$, which is the observed value of the random variable Z, when μ_τ and σ_τ are replaced by their estimates, $\hat{\mu}_\tau$ and $\hat{\sigma}_\tau$, respectively. Note that $z_{m,\tau}$ depends not only on the specified true moisture content, τ, but on m, the designated meter value, as well. $z_{m,\tau}$ is a number since m, $\hat{\mu}_\tau$, and $\hat{\sigma}_\tau$ are numbers.

After $z_{m,\tau}$ is computed, we need only to consult the standard normal tables in order to compute the probability in equation (4.17).

The decision concerning whether to discount for excessive moisture content depends on the meter reading. Corn will be graded No. 2 yellow corn if the moisture content as determined by the electronic moisture meter is less than or equal to 15.5 percent. If the meter reading is greater than that amount, the price that the farmer receives for the corn will be discounted. Thus, the corn will be discounted by mistake if the meter reads greater than 15.5 percent when the moisture content is indeed less than or equal to 15.5 percent. Therefore, the probability of corn being discounted when it should <u>not</u> be discounted is:

$$\Pr(M_T > 15.5 \mid T \leq 15.5).$$

Now, suppose we have a sample of corn with a true moisture content of $T = \tau$, where τ is some particular value that is less than or equal to 15.5 percent. Note that this corn meets the standards for U.S. Grade No. 2, and it should not be discounted for excessive moisture content. If the meter reads a value, M_τ, that is greater than 15.5 percent, then the corn will be discounted by mistake. We wish to know the probability that this will happen. Given that $T = \tau$, and applying equation (4.17), we have

Pr(corn with true moisture content equal to τ being discounted)

$$= \Pr(M_\tau > 15.5 \mid T = \tau)$$

$$= \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} > \frac{15.5 - \hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z > z_{15.5,\tau}).$$

Since Z is distributed as standard normal, we now need only to consult a standard normal table to obtain this probability.

Using this method, probabilities will be calculated at values of τ that are smaller than 15.5 percent. Since actual meter readings are in tenths, the selected values of τ will be at every one-tenth of a percentage point within a relevant range. We can then plot these probabilities against the values of τ. The resulting graphs will provide illustration of how this probability of incorrect discounting is related to true moisture content.

Similarly, we can determine the probability of corn not being discounted when it should be discounted, with respect to trade standards. This will happen if the electronic moisture meter reads a value less than or equal to 15.5 percent when the corn has a true moisture content greater than 15.5 percent. In other words, we are interested in

$$\Pr(M_T \leq 15.5 \mid T > 15.5).$$

Now suppose we have a sample of corn with the moisture content of $T = \tau$ and that τ is some value larger than 15.5 percent. This corn should be discounted for excessive moisture content, but it will not be if the meter reading, M_τ, is less than or equal to 15.5 percent. So, given $T = \tau$, the probability of not discounting when we should is equal to

$$\Pr(M_\tau \leq 15.5 \mid T=\tau) = \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} \leq \frac{15.5 - \hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z \leq z_{15.5,\tau}), \qquad (4.18)$$

where $Z \sim N(0, 1)$. This probability can now be obtained simply by consulting the standard normal table.

The probability in equation (4.18) will be computed for values of τ that are greater than 15.5 percent. The selected values of τ will be at every one-tenth of a percentage point within a relevant range. These probabilities will be plotted against the values of τ, with the resulting graphs providing illustration of the relationship between the probability of this type of incorrect discounting and true moisture content.

It is necessary to emphasize that we are dealing with conditional probabilities here. The probability distribution of meter measurement from which these probabilities are calculated depends on the selection of τ. Yet, by subtracting the mean and dividing by the standard deviation of the particular distribution of interest, we will always obtain the standard normal distribution and we can always determine the desired probability.

In fact, we need not restrict ourselves to considerations of only the probability of making an incorrect discount decision (discounting when we should not, or not discounting when we should). The method described in this section can be generalized easily so that we may compute the probability relating to any size of measurement error. All that is needed is a value for the true moisture content and either the error size or the meter reading of interest.

For instance, we may be interested in the probability that the meter will read a value larger than the true moisture content. That is, given that $T = \tau$, we could calculate the probability that M will be greater than τ:

$$\Pr(M_\tau > \tau \mid T = \tau).$$

Notice that, in this case, m of equation (4.17) is equal to τ. Subtracting the estimated mean of the distribution of M_τ and dividing by its standard deviation, we arrive at the observed value of Z, from which we can determine this probability:

$$\Pr(M_\tau > \tau | T=\tau) = \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} > \frac{\tau - \hat{\mu}_\tau}{\sigma_\tau}\right)$$

$$= \Pr(Z > z_{\tau,\tau}). \qquad (4.19)$$

A numerical example of equation (4.19) may be helpful to illustrate its use. Let τ = 20.0 percent. Then, the probability of the moisture meter reading a value greater than 20.0 percent would be calculated as follows:

$$\Pr(M_{20.0} > 20.0 | T=20.0) = \Pr\left(\frac{M_{20.0} - \hat{\mu}_{20.0}}{\hat{\sigma}_{20.0}} > \frac{20.0 - \hat{\mu}_{20.0}}{\hat{\sigma}_{20.0}}\right)$$

$$= \Pr(Z > z_{20.0,20.0}).$$

$M_{20.0}$ denotes the random variable representing the distribution of meter readings when the true moisture content is 20.0 percent. $\hat{\mu}_{20.0}$ and $\hat{\sigma}_{20.0}$ are the estimated mean and standard deviation, respectively, of that distribution. $z_{20.0,20.0}$ is the observed value of Z associated with these specifications, when m = 20.0 and τ = 20.0.

The probability in equation (4.19) will be calculated for values of τ, in tenths, within a relevant range. The results of the calculations will be plotted, as in the previous two cases.

We may also be interested in the probability that the measurement error will be larger (or smaller) than a certain size. Let E_τ be the random variable whose distribution is that of measurement error at T = τ, and ε be the designated error size. Then this probability is:

$$\Pr(E_\tau > \varepsilon | T=\tau) = \Pr(M_\tau > \tau+\varepsilon | T=\tau)$$

$$= \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} > \frac{(\tau+\varepsilon)-\hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z > z_{\tau+\varepsilon,\tau}). \qquad (4.20)$$

A numerical example of this situation might be if we were interested in the probability of the size of the measurement error being greater than, say, 1.0 percent. Then $\varepsilon = 1.0$ and the probability is calculated as follows:

$$\Pr(E_\tau > 1.0 | T=\tau) = \Pr(M_\tau > \tau+1.0 | T=\tau)$$

$$= \Pr\left(\frac{M_t - \hat{\mu}_\tau}{\hat{\sigma}_\tau} > \frac{(\tau+1.0)-\hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z > z_{\tau+1.0,\tau}).$$

Probabilities would be computed for values of τ, in tenths, within a relevant range. Specifically, if $\tau = 17.0$ percent, the probability of a measurement error larger than 1.0 percent would be

$$\Pr(E_{17.0} > 1.0 | T=17.0) = \Pr(M_{17.0} > 18.0 | T=17.0)$$

$$= \Pr\left(\frac{M_{17.0} - \hat{\mu}_{17.0}}{\hat{\sigma}_{17.0}} > \frac{18.0-\hat{\mu}_{17.0}}{\hat{\sigma}_{17.0}}\right)$$

$$= \Pr(Z > z_{18.0,17.0}) \qquad (4.21)$$

since $\tau + \varepsilon$ in equation (4.20) is equal to $17.0 + 1.0 = 18.0$.

In each of the previous examples, in order to calculate the desired probabilities, τ was, in a sense, an independent variable and allowed to vary. The value of m in equation (4.17) was fixed or depended on the value of τ. One final type of conditional probability may be calculated with τ fixed and m allowed to vary. We may ask this question: Given a sample of corn with true moisture content, $T = \tau$, what is the probability that the meter will read a value greater than specified values of m? For example, if $\tau = 15.5$ percent, then this probability is

$$\Pr(M_{15.5} > m | T=15.5) = \Pr\left(\frac{M_{15.5}-\hat{\mu}_{15.5}}{\hat{\sigma}_{15.5}} > \frac{m-\hat{\mu}_{15.5}}{\hat{\sigma}_{15.5}}\right)$$

$$= \Pr(Z > z_{m,15.5}),$$

and would be calculated for values of m, in tenths, within a relevant range. In particular, given that $\tau = 15.5$, the probability that the meter reading will be greater than, say, $m = 12.0$ percent, is

$$\Pr(M_{15.5} > 12.0 | T=15.5) = \Pr\left(\frac{M_{15.5}-\hat{\mu}_{15.5}}{\hat{\sigma}_{15.5}} > \frac{12.0-\hat{\mu}_{15.5}}{\hat{\sigma}_{15.5}}\right)$$

$$= \Pr(Z > z_{12.0,15.5}).$$

Ideally, a graph of the resulting probabilities of this type, for any $T = \tau$, should look like the one in Figure 4.3.

For example, the probability that the meter reads greater than 12.0 percent when the true moisture content is, indeed, 15.5 percent, should be one:

$$\Pr(M_{15.5} > 12.0 | T=15.5) = 1.$$

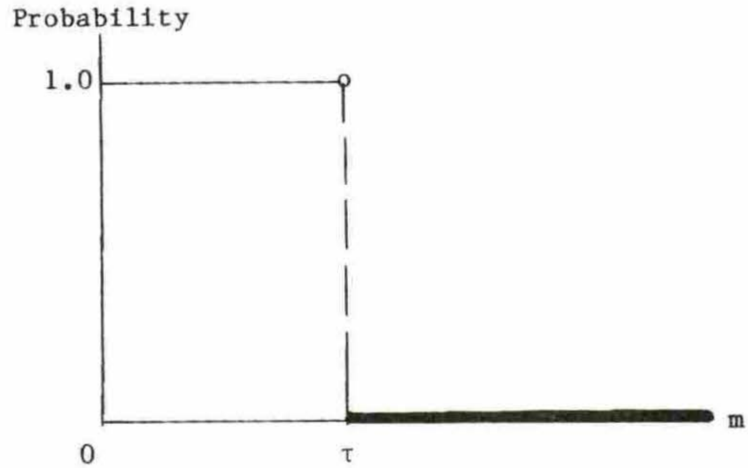


Figure 4.3. Ideal graph of probabilities of the form $Pr(M_\tau > m | T=\tau)$ with $T = \tau$ fixed

But the probability of the meter reading being greater than 20.0 percent when $\tau = 15.5$ percent should be zero:

$$Pr(M_{15.5} > 20.0 | T=15.5) = 0.$$

Finally, notice that the observed value of Z depends on both the given true moisture content, τ, and the chosen meter reading, m, or error size, ε, of interest. For instance, consider the example in equation (4.21), $Pr(E>1.0|T=17.0)$. Suppose that we also want to know $Pr(E_{17.0}>0.5|T=17.0)$. The true moisture content remains the same in both problems, but the designated error size has changed. Therefore, the observed values of Z will be different in the two problems. In the second problem, $z_{17.5,17.0} = \dfrac{(17.0+0.5)-\hat{\mu}_{17.0}}{\hat{\sigma}_{17.0}}$, which is clearly not equal to $z_{18.0,17.0} = \dfrac{(17.0+1.0)-\hat{\mu}_{17.0}}{\hat{\sigma}_{17.0}}$. The notation for the observed value of Z serves to emphasize its relation to both the true moisture, τ, and either m or ε.

In summary, the procedure outlined in this section is versatile and can be applied in calculating several types of conditional probabilities of meter error in measuring moisture content in corn. One needs only to specify the values of m and τ in equation (4.17) in accord with the question being asked.

5. THE RESULTS

5.1. Regression Results

5.1.1. Estimated variance functions

The model in equation (4.2), which describes the relationship between the sample variances, V^s, and true moisture content, T, was estimated by the ordinary least squares methods described in section 4.2.1 for each moisture meter for each period. Both linear and quadratic forms were estimated. Since the variance function was estimated as a means of testing for heteroscedasticity in equation (4.3), the t-statistics for significance of the parameter estimates, especially for the slope parameter, are relevant. The R^2 value, which gives the proportion of variation in V^s attributable to the model, was also considered. For this study, the greater the amount of variation explained, the better, since the estimated variances from this function are used in the generalized least squares (GLS) estimation in section 4.2.2. Both the t-statistics and the R^2 values were judged in order to choose the better functional form of the model in each period.

The estimation results for the Burrows, GAC II, Motomco, and Steinlite meters are summarized in Tables 5.1, 5.2, 5.3, and 5.4, respectively. All estimated equations were significant at the 0.1 percent level. The functional form selected as the more appropriate is noted by reference to a footnote.

The selected variance functions for each meter and each period were then used to provide estimates of the elements of the V-matrix, the

Table 5.1. Estimated variance functions for the Burrows meter

Burrows	Linear model		Quadratic model	
1979	$\hat{V} = -0.135 - 0.00964T$ (-7.55)[b] (11.19)	$R^2 = 0.172$	$\hat{V} = 0.193 - 0.0219T + 0.000694T^2$ [a] (-3.50) (-4.29) (6.26)	$R^2 = 0.223$
1981-82	$\hat{V} = -0.135 + 0.00936T$ (-11.17) (18.12)	$R^2 = 0.250$	$\hat{V} = 0.0721 - 0.0100T + 0.000420T^2$ [a] (1.74) (-2.67) (5.21)	$R^2 = 0.270$

[a]The selected model of the variance function.

[b]t-statistics for significance of parameter estimates.

Table 5.2. Estimated variance functions for the GAC II meter

GAC II	Linear model	Quadratic model
1979	$\hat{V} = -0.131 + 0.00923T$ $(-10.60)^{b}$ (15.50) $R^2 = 0.287$	$\hat{V} = 0.207 - 0.0231T + 0.000709T^2$ [a] (5.62) (-6.82) (9.67) $R^2 = 0.384$
1981-82	$\hat{V} = -0.130 + 0.00914T$ (-12.18) (20.06) $R^2 = 0.167$	$\hat{V} = 0.174 - 0.0216T + 0.000716T^2$ [a] (6.09) (-7.91) (11.41) $R^2 = 0.217$

[a] The selected model of the variance function.

[b] t-statistics for significance of parameter estimates.

Table 5.3. Estimated variance functions for the Motomco meter

Motomco	Linear model	Quadratic model
1979	$\hat{V} = -0.127 + 0.00931T$ $(-10.52)^{b}$ (16.07) $R^2 = 0.302$	$\hat{V} = 0.0827 - 0.0107T + 0.000438T^2$ [a] (2.21) (-3.13) (5.93) $R^2 = 0.341$
1981-82	$\hat{V} = -0.154 + 0.0109T$ (-11.65) (18.65) $R^2 = 0.147$	$\hat{V} = 0.127 - 0.0172T + 0.000652T^2$ [a] (3.43) (-4.92) (8.16) $R^2 = 0.174$

[a]The selected model of the variance function.

[b]t-statistics for significance of parameter estimates.

Table 5.4. Estimated variance functions for the Steinlite meter

Steinlite	Linear model	Quadratic model
1979	$\hat{V} = -0.0464 + 0.00404T$ [a] (-7.15)[b] (12.83) $R^2 = 0.219$	$\hat{V} = 0.00916 - 0.00132T + 0.000119T^2$ (0.45) (-0.70) (2.87) $R^2 = 0.229$
1981-82	$\hat{V} = -0.0901 + 0.00623T$ (-11.85) (19.85) $R^2 = 0.159$	$\hat{V} = 0.0897 - 0.0118T + 0.000419T^2$ [a] (4.45) (-6.21) (9.61) $R^2 = 0.196$

[a]The selected model of the variance function.

[b]t-statistics for significance of parameter estimates.

variance-covariance matrix of the error term of the model in equation (4.3), which related the meter measure, M, and T. Examples of the calculations follow.

The estimated variance of the meter measure given that T = 14.7 percent, for the Steinlite meter in 1979, is

$$\hat{v}_{14.7} = -0.0464 + 0.00404(14.7)$$

$$= 0.01307. \quad (5.1)$$

For the Motomco meter in 1981-82, this estimated variance, when T = 15.9 percent, is

$$\hat{v}_{15.9} = 0.127 - 0.0172(15.9) + 0.000652(15.9)^2$$

$$= 0.01805. \quad (5.2)$$

If T = 20.0 percent, then the estimated variance of the meter measure, for the GAC II meter in 1979, is

$$\hat{v}_{20.0} = 0.207 - 0.0231(20.0) + 0.000709(20.0)^2$$

$$= 0.02834. \quad (5.3)$$

5.1.2. Estimates of M = g(T)

The function M = g(T), relating the meter measure and the true value of moisture content, was described by the statistical model in equation (4.6), and was estimated by the generalized least squares techniques explained in section 4.2.2. Both linear and quadratic forms were estimated. The t-statistics for significance of the parameter estimates, and the standard errors of the estimates were considered in choosing the more appropriate functional form.

In addition, since the usual reported R^2 value is not relevant in GLS regressions such as this one, two other goodness-of-fit criteria were considered. One of these was the mean-squared error, MSE, of the regression. This is defined to be the error sum of squares (ESS) divided by the number of error degrees of freedom, and in GLS models, it is an estimate of a scalar multiplier of the variance of the error term. Smaller values of MSE are preferred to larger ones.

Furthermore, an alternative to the usual R^2 goodness-of-fit measure in GLS regressions is the square of the simple correlation coefficient between the actual and predicted values, denoted $r^2_{M,\hat{M}}$. This will be calculated as well. The closer is $r^2_{M,\hat{M}}$ to one, the better.

The estimation results for each period are summarized in Tables 5.5, 5.6, 5.7, and 5.8 for the Burrows, GAC II, Motomco, and Steinlite meters, respectively. All estimated equations were significant at the 0.1 percent level. The t-statistics, MSE and $r^2_{M,\hat{M}}$, are reported for each estimated model. The functional form judged to be the more appropriate is noted by reference to a footnote.

For each meter, the selected form of $M = g(T)$ will be used to estimate the mean of the distribution of meter measurements at a given value of T. Example calculations of $\hat{M}$ follow.

Table 5.5. Estimates of $M = g(T)$ for the Burrows meter

Burrows	Linear model	Quadratic model
1979	$\hat{M} = -0.711 + 1.0729T$ (-5.26)[b] (141.01) MSE = 48.0679 $r^2_{M,\hat{M}} = 0.970$	$\hat{M} = -1.530 + 1.160T - 0.00221T^2$ [a] (-3.45) (25.37) (-1.94) MSE = 47.85 $r^2_{M,\hat{M}} = 0.972$
1981-82	$\hat{M} = -2.154 + 1.0966T$ [a] (-18.04) (170.60) MSE = 74.686 $r^2_{M,\hat{M}} = 0.962$	$\hat{M} = -1.389 + 1.0176T + 0.00189T^2$ (-3.06) (22.33) (1.75) MSE = 74.530 $r^2_{M,\hat{M}} = 0.961$

[a]The selected model of $M = g(T)$.

[b]t-statistics for significance of parameter estimates.

Table 5.6. Estimates of M = g(T) for the GAC II meter

GAC II	Linear model		Quadratic model	
1979	$\hat{M} = -0.413 + 1.0325T$ $(-3.45)^{b}$ (154.45)		$\hat{M} = -1.683 + 1.167T - 0.00340T^{2}$ [a] (-4.32) (29.25) (-3.43)	
	MSE = 39.127	$r^2_{M,\hat{M}} = 0.974$	MSE = 38.438	$r^2_{M,\hat{M}} = 0.977$
1981-82	$\hat{M} = -1.670 + 1.0811T$ (-21.42) (263.46)		$\hat{M} = 0.740 + 0.821T + 0.00656T^{2}$ [a] (2.84) (30.23) (9.68)	
	MSE = 65.933	$r^2_{M,\hat{M}} = 0.967$	MSE = 63.031	$r^2_{M,\hat{M}} = 0.970$

[a]The selected model of M = g(T).

[b]t-statistics for significance of parameter estimates.

Table 5.7. Estimates of $M = g(T)$ for the Motomco meter

Motomco	Linear model		Quadratic model	
1979	$\hat{M} = 0.502 + 0.961T$ $(3.90)^{b}$ (129.15) MSE = 48.73	$r^2_{M,\hat{M}} = 0.963$	$\hat{M} = -1.598 + 1.189T - 0.00576T^2$ [a] (-4.01) (28.61) (-5.56) MSE = 46.40	$r^2_{M,\hat{M}} = 0.970$
1981-82	$\hat{M} = -1.512 + 1.0529T$ (-22.22) (288.47) MSE = 49.124	$r^2_{M,\hat{M}} = 0.966$	$\hat{M} = 0.127 + 0.869T + 0.00474T^2$ (0.60) (38.08) (8.16) MSE = 47.579	$r^2_{M,\hat{M}} = 0.965$
			$\hat{M} = 0.883T + 0.00441T^2$ [a] (221.36) (24.01) MSE = 47.564	$r^2_{M,\hat{M}} = 0.965$

[a]The selected model of $M = g(T)$.

[b]t-statistics for significance of parameter estimates.

Table 5.8. Estimates of $M = g(T)$ for the Steinlite meter

Steinlite	Linear model	Quadratic model
1979	$\hat{M} = 1.363 + 0.967T$ (11.34)[b] (130.27) MSE = 99.148 $r^2_{M,\hat{M}} = 0.976$	$\hat{M} = -1.948 + 1.336T - 0.00944T^2$ [a] (-5.18) (32.86) (-9.23) MSE = 86.740 $r^2_{M,\hat{M}} = 0.983$
1981-82	$\hat{M} = -1.452 + 1.0516T$ [a] (-21.38) (288.03) MSE = 87.258 $r^2_{M,\hat{M}} = 0.976$	$\hat{M} = -1.114 + 1.014T + 0.000956T^2$ (-4.97) (42.43) (1.58) MSE = 87.192 $r^2_{M,\hat{M}} = 0.976$

[a]The selected model of $M = g(T)$.

[b]t-statistics for significance of parameter estimates.

Let T = 14.7 percent. The expected value of meter moisture content, $\hat{M}$, for the Steinlite meter in 1979 is

$$\hat{M}_{14.7} = -1.948 + 1.336(14.7) - 0.00944(14.7)^2$$

$$= 15.65. \tag{5.4}$$

For the Motomco meter in 1981-82, the estimated mean meter reading when T = 15.9 percent is

$$\hat{M}_{15.9} = 0.883(15.9) + 0.00441(15.9)^2$$

$$= 15.15. \tag{5.5}$$

If the true moisture content is T = 20.0 percent, then the expected meter reading for the GAC II meter in 1979 is

$$\hat{M}_{20.0} = -1.683 + 1.167(20.0) - 0.00340(20.0)^2$$

$$= 20.30. \tag{5.6}$$

5.1.3. Results of the test for equality of parameters

Equation (4.9) gives the formula for the F-statistic to be used in testing the estimated parameters from the two periods for equality. The null hypothesis is $H_0 : \beta_1 = \beta_2$, where the subscript refers to the period. A significant observed F-statistic at the five percent level results in rejection of H_0, supporting the conclusion that the models for the two periods are significantly different.

Notice that $\beta_1 = (\beta_{11}\ \beta_{12}\ \beta_{13})'$ and $\beta_2 = (\beta_{21}\ \beta_{22}\ \beta_{23})'$ if the models are quadratic. Then the null hypothesis can be rewritten as

$H_0 : \beta_{11} = \beta_{21}$ <u>and</u> $\beta_{12} = \beta_{22}$ <u>and</u> $\beta_{13} = \beta_{23}$. This is an "and" statement. For the entire statement to be true, all three component statements must be true simultaneously. Thus, if $\beta_{21} = 0$ and β_{11} is significantly different from zero, then β_1 is significantly different from β_2, which infers that the models from the two periods are not the same.

For example, consider the estimated equations for the Motomco meter provided in Table 5.8. In 1979, the selected model is quadratic, with all three coefficients significant. But, the selected model for 1981-82 is quadratic with $\beta_{21} = 0$. Thus, logically, β_1 is significantly different from β_2, and H_0 is rejected for the Motomco meter. The F-statistic was not calculated for this meter.

Similar situations occur for the Burrows and Steinlite meters, where a quadratic model is selected for the 1979 data and a linear model is selected for 1981-82. For each of these meters, $\beta_{23} = 0$, but β_{13} is significantly different from zero. Thus, H_0 is rejected for these two meters, as well. For the sake of curiosity, though, the F-statistics comparing the quadratic models for these meters were calculated.

The observed F-statistics for the Burrows, GAC II, and Steinlite meters and the conclusions of the test are reported in Table 5.9. For all four meters, then, whether tested logically or statistically, we conclude that the parameter estimates from the two periods are significantly different from each other. Thus, for each meter, the model of $M = g(T)$ for 1979 data is not the same as the model for 1981-82 data. Recalibration of the moisture meters was effective in changing the relationship between meter measures of moisture content and the true value.

Table 5.9. Results of the F-test for equality of parameter estimates

Meter	F_{obs}	$Prob(F>F_{obs})$	Conclusion
Burrows	509.39	0	Reject H_0
GAC II	205.26	0	Reject H_0
Steinlite	1836.3	0	Reject H_0

5.2. The Probability Distributions of Meter Measurement

5.2.1. Estimates of the individual means and variances

Equations (4.15) and (4.16) give the formulas to be used for calculating estimates of the mean and the variance of the individual probability distribution of M_τ, where $M_\tau \sim N(\mu_\tau, \sigma_\tau^2)$ for a particular value of true moisture content, $T = \tau$. These equations are repeated here for convenience:

$$\hat{\mu}_\tau = \hat{M}_\tau = T_\tau' \hat{b}^G \tag{5.7}$$

$$\hat{\sigma}_\tau^2 = T_\tau' D(\hat{b}^G) T_\tau + \hat{v}_\tau. \tag{5.8}$$

T_τ' is a row vector representing $T = \tau$, $\hat{b}^G$ contains the parameter estimates of $M = g(T)$, $D(\hat{b}^G)$ is the dispersion matrix associated with $\hat{b}^G$, and $\hat{v}_\tau$ is the value of the variance function at $T = \tau$.

The selected estimated equations for $\hat{M}$ and $\hat{V}$, and the dispersion matrices, for each meter are summarized in Tables 5.10(a) and (b) for 1979 and 1981-82, respectively. We now have all the information necessary to calculate individual means and variances at any value of T.

For example, let T = 14.7 percent. For the Steinlite meter in 1979, the mean $\hat{\mu}_{14.7}$ is

$$\hat{\mu}_{14.7} = \hat{M}_{14.7} = 15.65,$$

which was found in equation (5.4). The variance is

$$\hat{\sigma}^2_{14.7} = T'_{14.7} D(\hat{b}^G) T_{14.7} + \hat{v}_{14.7}$$

$$= (1 \ 14.7 \ 14.7^2) \begin{pmatrix} 1.6\text{x}10^{-3} & -1.7\text{x}10^{-4} & 4.2\text{x}10^{-6} \\ & 1.9\text{x}10^{-5} & -4.7\text{x}10^{-7} \\ \text{symm.} & & 1.2\text{x}10^{-8} \end{pmatrix} \begin{pmatrix} 1 \\ 14.7 \\ 14.7^2 \end{pmatrix} + \hat{v}_{14.7}$$

$$= 0.00001 + 0.01307$$

$$= 0.01308,$$

where $\hat{v}_{14.7}$ was found in equation (5.1). Thus, $M_{14.7} \sim N(15.65, 0.01308)$ for the Steinlite meter in 1979.

Let T = 15.9 percent and $T'_{15.9} = (15.9 \ 15.9^2)$. Then, for the Motomco meter in 1981-82,

$$\hat{\mu}_{15.9} = \hat{M}_{15.9} = 15.15$$

Table 5.10. Summary of estimated equations and dispersion matrices

a) 1979

Meter		Estimated equations	$D(\hat{b}^G)$
Burrows	$\hat{V}$	$\hat{V} = 0.193 - 0.0219T + 0.000694T^2$	$\begin{pmatrix} 4.1 \times 10^{-3} & -4.2 \times 10^{-4} & 1.0 \times 10^{-5} \\ & 4.4 \times 10^{-5} & -1.1 \times 10^{-6} \\ \text{symm.} & & 2.7 \times 10^{-8} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -1.530 + 1.160T - 0.00221T^2$	
GAC II	$\hat{V}$	$\hat{V} = 0.207 - 0.0231T + 0.000709T^2$	$\begin{pmatrix} 3.9 \times 10^{-3} & -4.0 \times 10^{-4} & 9.6 \times 10^{-6} \\ & 4.1 \times 10^{-5} & -1.0 \times 10^{-6} \\ \text{symm.} & & 2.6 \times 10^{-8} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -1.683 + 1.167T - 0.00340T^2$	
Motomco	$\hat{V}$	$\hat{V} = 0.0827 - 0.0107T + 0.000438T^2$	$\begin{pmatrix} 3.4 \times 10^{-3} & -3.5 \times 10^{-4} & 8.4 \times 10^{-6} \\ & 3.7 \times 10^{-5} & -9.1 \times 10^{-7} \\ \text{symm.} & & 2.3 \times 10^{-8} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -1.598 + 1.189T - 0.00576T^2$	
Steinlite	$\hat{V}$	$\hat{V} = -0.0464 + 0.00404T$	$\begin{pmatrix} 1.6 \times 10^{-3} & -1.7 \times 10^{-4} & 4.2 \times 10^{-6} \\ & 1.9 \times 10^{-5} & -4.7 \times 10^{-7} \\ \text{symm.} & & 1.2 \times 10^{-8} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -1.948 + 1.336T - 0.00944T^2$	

b) 1981-82

Meter		Estimated equations	$D(\hat{b}^G)$
Burrows	$\hat{V}$	$\hat{V} = 0.0721 - 0.0100T + 0.000420T^2$	$\begin{pmatrix} 1.9 \times 10^{-4} & -9.9 \times 10^{-6} \\ \text{symm.} & 5.5 \times 10^{-7} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -2.154 + 1.0966T$	
GAC II	$\hat{V}$	$\hat{V} = 0.174 - 0.0216T + 0.000716T^2$	$\begin{pmatrix} 1.1 \times 10^{-3} & -1.1 \times 10^{-4} & 2.2 \times 10^{-6} \\ & 1.2 \times 10^{-5} & -2.9 \times 10^{-7} \\ \text{symm.} & & 7.3 \times 10^{-9} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = 0.740 + 0.821T + 0.00656T^2$	
Motomco	$\hat{V}$	$\hat{V} = 0.127 - 0.0172T + 0.000652T^2$	$\begin{pmatrix} 3.3 \times 10^{-7} & -1.5 \times 10^{-8} \\ \text{symm.} & 7.1 \times 10^{-10} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = 0.883T + 0.00441T^2$	
Steinlite	$\hat{V}$	$\hat{V} = 0.0897 - 0.0118T + 0.000419T^2$	$\begin{pmatrix} 5.3 \times 10^{-5} & -2.7 \times 10^{-6} \\ \text{symm.} & 1.5 \times 10^{-7} \end{pmatrix}$
	$\hat{M}$	$\hat{M} = -1.452 + 1.0516T$	

from equation (5.5), and

$$\hat{\sigma}^2_{15.9} = T'_{15.9}D(\hat{b}^G)T_{15.9} + \hat{v}_{15.9}$$

$$= 0.00001 + 0.01805$$

$$= 0.01806,$$

where $D(\hat{b}^G)$ can be found in Table 5.10 and $\hat{v}_{15.9}$ was found in equation (5.2). Thus, $M_{15.9} \sim N(15.15, 0.01806)$ for the Motomco meter in 1981-82.

Finally, in 1979, for the GAC II meter, when T = 20.0 and $T'_{20.0} = (1\ 20\ 20^2)$,

$$\hat{\mu}_{20.0} = \hat{M}_{20.0} = 20.30,$$

found in equation (5.6), and

$$\hat{\sigma}^2_{20.0} = T'_{20.0}D(\hat{b}^G)T_{20.0} + \hat{v}_{20.0}$$

$$= 0.02837,$$

where $D(\hat{b}^G)$ is given in Table 5.10 and $\hat{v}_{20.0}$ was found in equation (5.3). $M_{20.0} \sim N(20.30, 0.02837)$ for the GAC II meter in 1979.

5.2.2. Probability calculations

Now that we have estimtes of μ_τ and σ^2_τ for any value of true moisture content, $T = \tau$, we can use equation (4.17) to calculate the probability of various types of measurement error. That is, given that $T = \tau$, the probability that the corresponding meter measurement is greater than or less than a specified value of, say, m, is

$$\Pr(M_\tau \gtrless m \mid T=\tau) = \Pr\left(\frac{M_\tau - \hat{\mu}_\tau}{\hat{\sigma}_\tau} \gtrless \frac{m-\hat{\mu}_\tau}{\hat{\sigma}_\tau}\right)$$

$$= \Pr(Z \gtrless z_{m,\tau}) \tag{5.9}$$

This formula may be used to calculate probabilities of different types of error for the various meters. Examples of calculations follow.

Suppose we have a sample of corn with true moisture content of 14.7 percent that was tested for moisture content using the Steinlite meter in 1979. The probability that the meter read a value greater than 15.5 percent was:

$$\Pr(M_{14.7} > 15.5 \mid T=14.7) = \Pr\left(\frac{M_{14.7} - \hat{\mu}_{14.7}}{\hat{\sigma}_{14.7}} > \frac{15.5-\hat{\mu}_{14.7}}{\hat{\sigma}_{14.7}}\right)$$

$$= \Pr(Z > \frac{15.5-15.65}{0.114})$$

$$= \Pr(Z > -0.917)$$

$$= 0.821, \tag{5.10}$$

where $\hat{\mu}_{14.7}$ and $\hat{\sigma}^2_{14.7}$ were found in section 5.2.1. Thus, there is an 82.1 percent chance that the meter will read a value greater than 15.5 percent when the corn has a true moisture content of 14.7 percent. Notice that this corn would have been discounted when it should not have been.

Suppose a sample of corn with a true moisture content of 15.9 percent is to be tested using a Motomco meter. Since the meter calibrations of 1981-82 are current, the probability that the meter will read a value less than 15.5 percent is

$$\Pr(M_{15.9} \leq 15.5 | T=15.9) = \Pr\left(\frac{M_{15.9}-\hat{\mu}_{15.9}}{\hat{\sigma}_{15.9}} \leq \frac{15.5-\hat{\mu}_{15.9}}{\hat{\sigma}_{15.9}}\right)$$

$$= \Pr(Z \leq \frac{15.5-15.15}{0.134})$$

$$= \Pr(Z \leq 2.986)$$

$$= 0.999, \tag{5.11}$$

where $\hat{\mu}_{15.9}$ and $\hat{\sigma}^2_{15.9}$ were found in section 5.2.1. The probability is 99.9 percent that the meter will read a value less than 15.5 percent when the corn has true moisture content of 15.9 percent. Notice that this corn would not be discounted for excessive moisture content when it should be.

One final illustration calculates the probability that the meter would read a value greater than a true moisture content of T = 20.0 percent for the GAC II meter in 1979:

$$\Pr(M_{20.0} > 20.0 | T=20.0) = \Pr\left(\frac{M_{20.0}-\hat{\mu}_{20.0}}{\hat{\sigma}_{20.0}} > \frac{20.0<\hat{\mu}_{20.0}}{\hat{\sigma}_{20.0}}\right)$$

$$= \Pr(Z > \frac{20.0-20.30}{0.168})$$

$$= \Pr(Z > -1.51)$$

$$= 0.934. \tag{5.12}$$

These methods will be used in the next section to calculate the probabilities of various types of error.

5.3. Probabilities of Error

5.3.1. The probability of discounting when should not

A sample of corn with a true moisture content of 15.5 percent or less meets the moisture content standards for No. 2 corn and should not be discounted for excessive moisture content. But, it will be discounted if the electronic moisture meter used for testing its moisture content gives a reading greater than 15.5 percent. For a particular value of $T = \tau$, where $\tau \leq 15.5$, the probability of this sample of corn being discounted when it should not be is

$$\Pr(M_{\tau} > 15.5 \mid T = \tau).$$

The methods in section 5.2 can be used to calculate this probability for relevant values of T.

For example, consider a sample of corn with a true moisture content of 14.7 percent, which should not be discounted for excessive moisture. If this sample were tested by a Steinlite meter in 1979, the probability that it would have been discounted, from equation (5.10), is

$$\Pr(M_{14.7} > 15.5 \mid T = 14.7) = 0.821.$$

There was an 82.1 percent chance that this corn would have been wrongly discounted due to meter error in measurement.

If this same sample were tested by the same Steinlite meter after 1981, after recalibration of the meters, the probability that it would be discounted when it should not be is

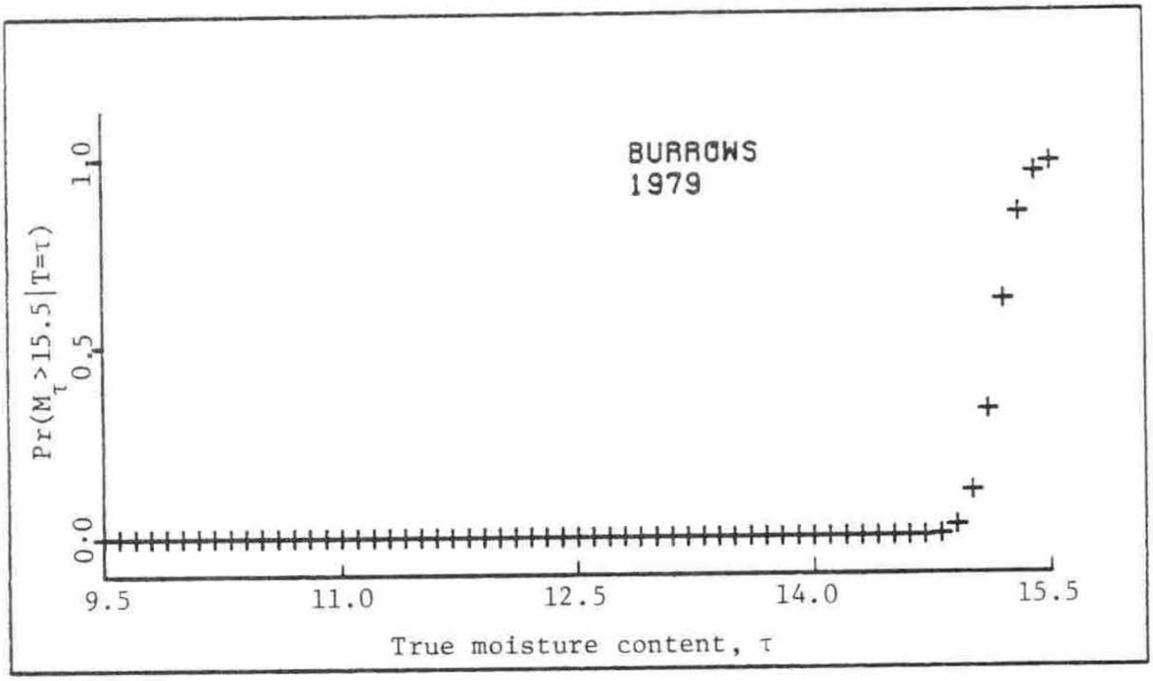
$$\Pr(M_{14.7}>15.5|T=14.7) = 0.0.$$

Similar calculations were carried out for values of T at every one-tenth of a percentage point between 9.5 percent and 15.5 percent. The resulting probabilities were plotted against true moisture content. This was done for each meter and for each period. These graphs are displayed in Figures 5.1, 5.2, 5.3, and 5.4 for the Burrows, GAC II, Motomco, and Steinlite meters, respectively. Each figure contains the graphs for both periods in order to facilitate easy visual comparison of the results, before and after recalibration.

Let us look at these graphs for each meter individually. Consider first the Burrows meter. For some value of the moisture content smaller than 15.5 percent, the meter begins to read a value larger than 15.5 percent with a probability greater than zero. The relevant values of true moisture content, $T = \tau$, and the resulting probabilities for each period are displayed in Table 5.11.

Table 5.11. Burrows: Selected probabilities of discounting corn that should not be discounted

Burrows	$\Pr(M_\tau>15.5\|T=\tau)$	
τ	1979	1981-82
14.9	0.027	0.0
15.0	0.119	0.0
15.1	0.335	0.0
15.2	0.629	0.0
15.3	0.862	0.0
15.4	0.968	0.0
15.5	0.995	0.0



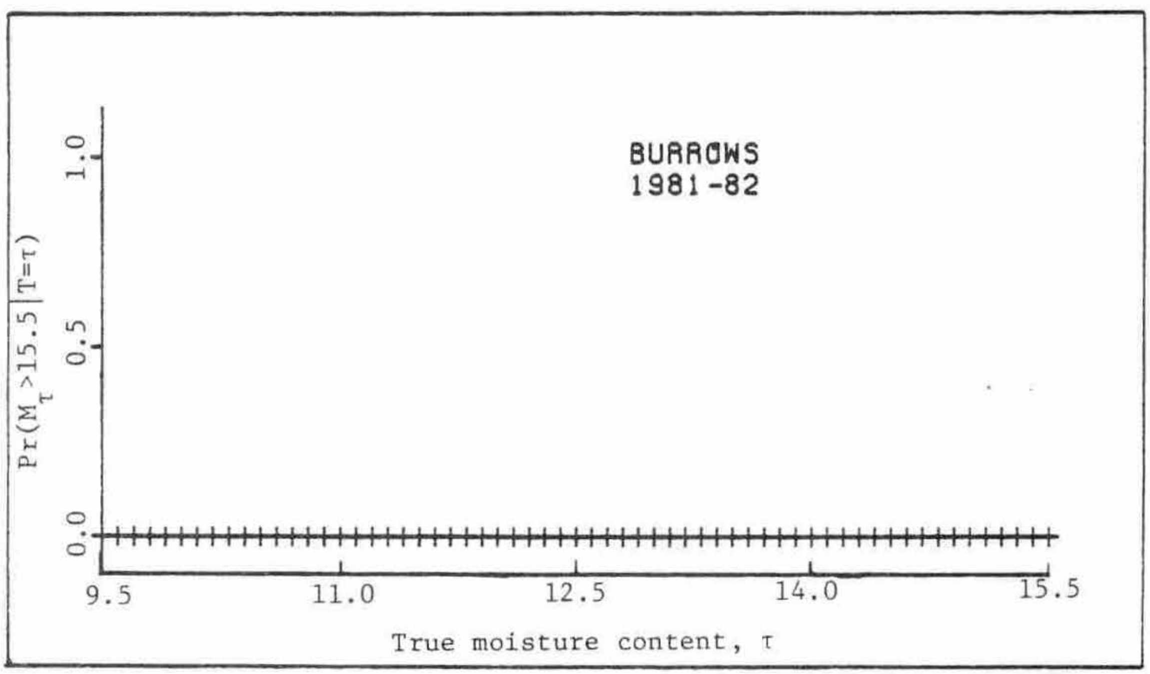


Figure 5.1. Burrows: Probabilities of discounting corn that should not be discounted

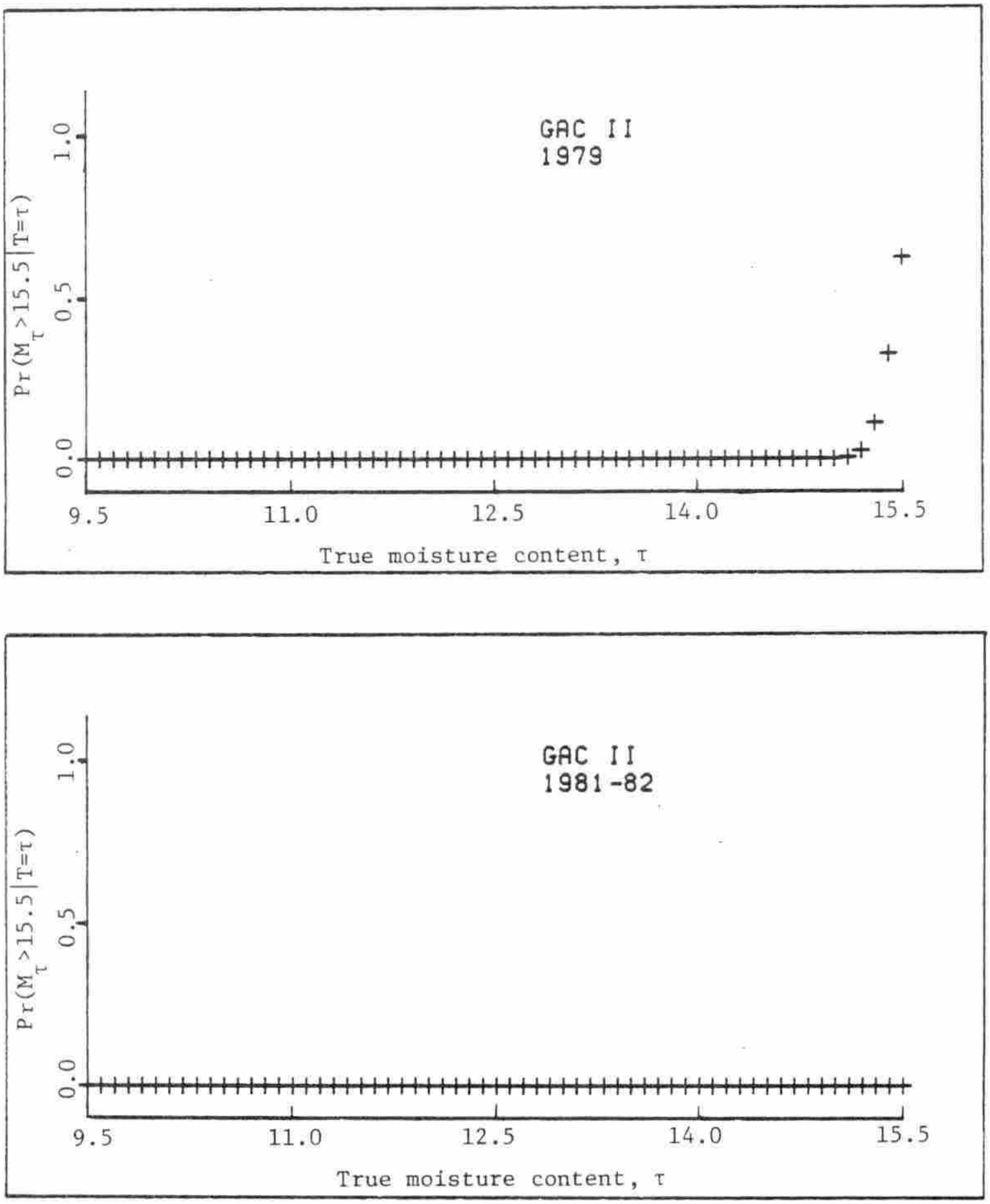
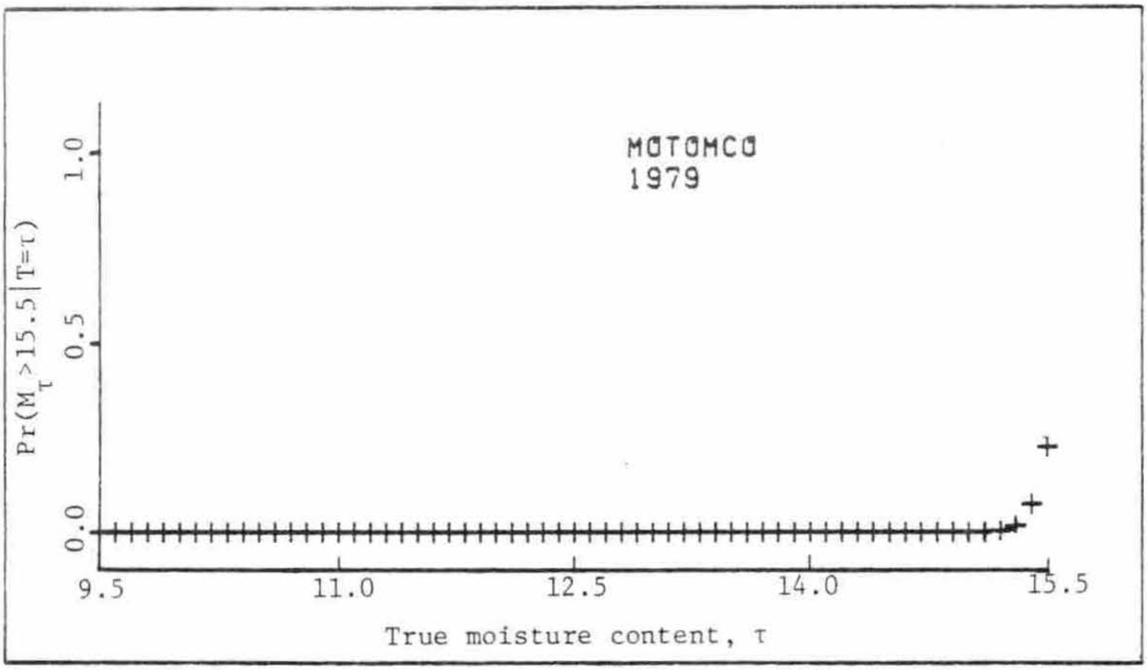


Figure 5.2. GAC II: Probabilities of discounting corn that should not be discounted



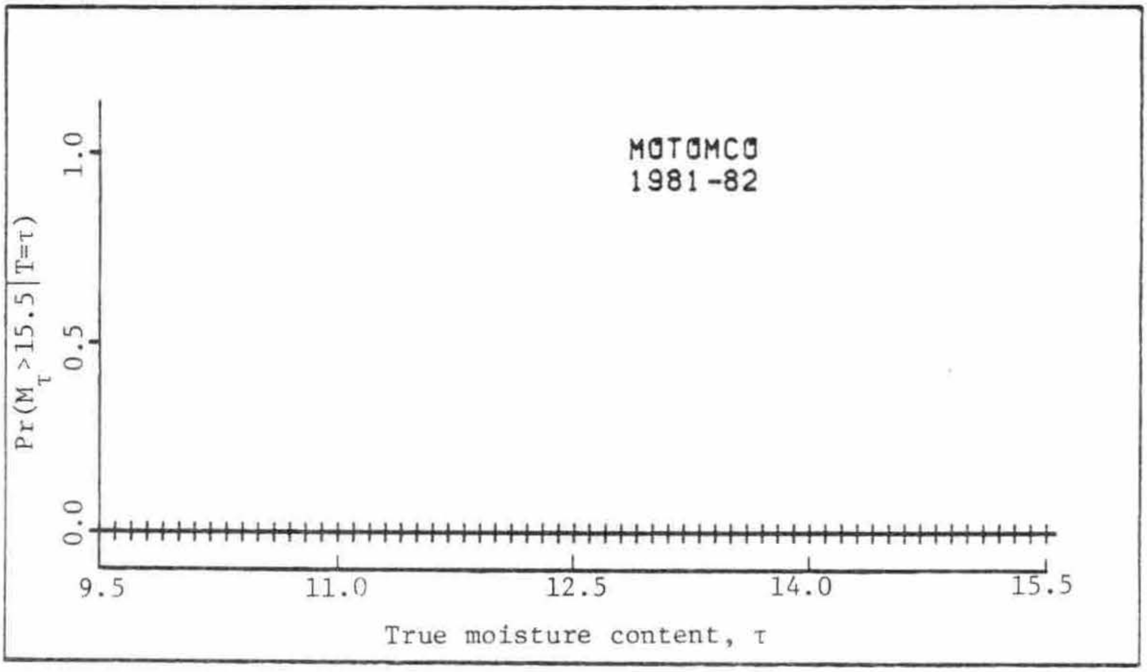
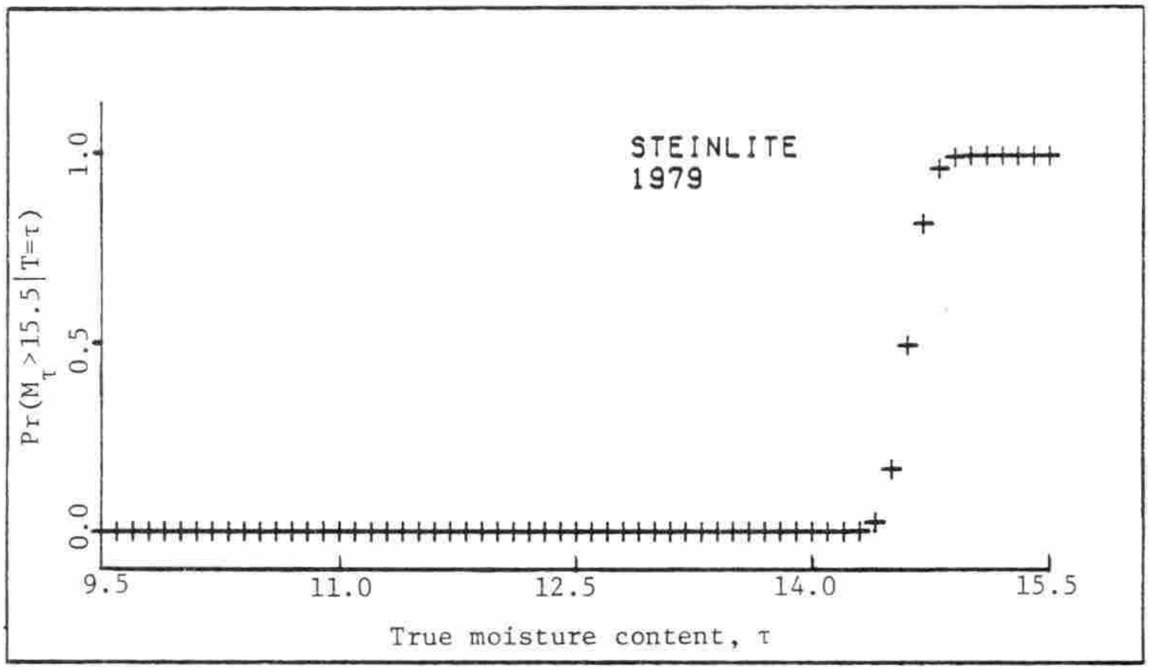


Figure 5.3. Motomco: Probabilities of discounting corn that should not be discounted



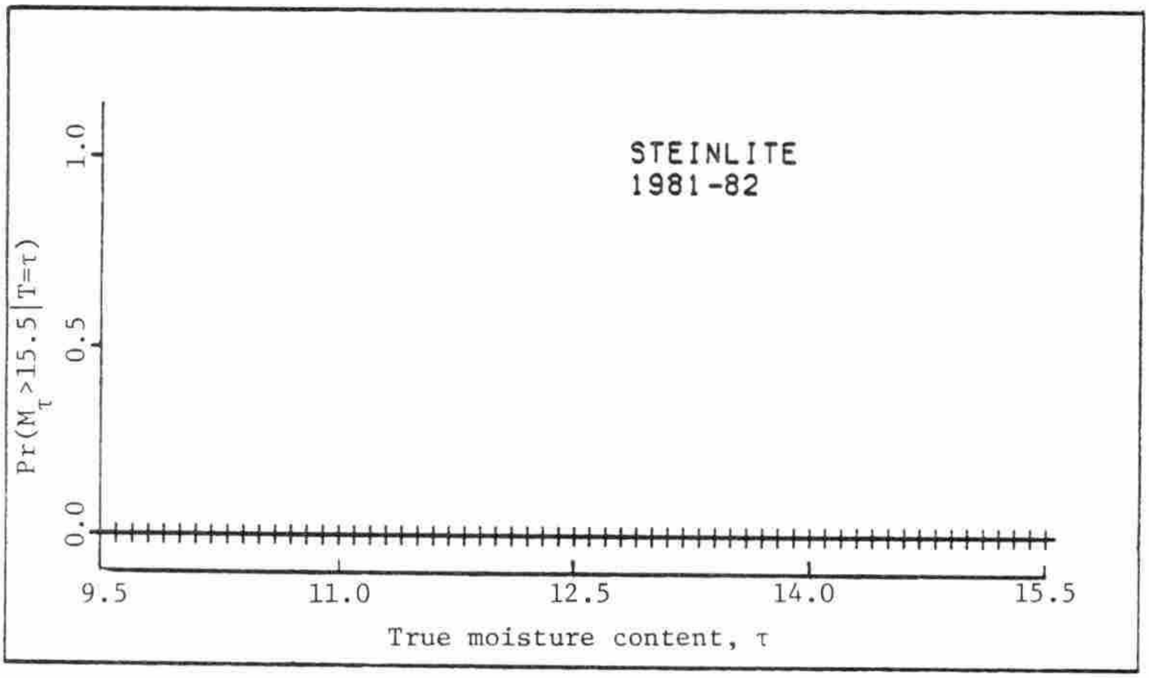


Figure 5.4. Steinlite: Probabilities of discounting corn that should not be discounted

Suppose, in 1979, a farmer delivered a load of corn with a true moisture content of 15.5 percent to a country elevator using a Burrows moisture meter to test for moisture content. This corn met the moisture content standards for grade No. 2. But, there was a probability of nearly one that this corn would be discounted for excessive moisture. In fact, if the corn had a true moisture content of 15.3 percent, there was a probability of 86.2 percent that the Burrows meter would give a reading greater than 15.5 percent, and thus cause the elevator manager to discount the price of the corn. However, since the meters were recalibrated, these probabilities are zero.

Suppose the country elevators were using a GAC II meter. Then, in 1979, there was, at worst, a probability of 62.6 percent that corn with a true moisture content of 15.5 percent would be discounted. For values of T smaller than that, there was much less than a 50-50 chance of improper discounting. These results are shown in Table 15.12. Notice that after recalibration, all probabilities are zero.

Table 5.12. GAC II: Selected probabilities of discounting corn that should not be discounted

GAC II	$Pr(M_\tau > 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.2	0.024	0.0
15.3	0.113	0.0
15.4	0.327	0.0
15.5	0.626	0.0

Consider now the Motomco meter. In 1979, at its worst, this meter would result in improper discounting of corn with true moisture content of 15.5 percent only 22.6 percent of the time. Other values of T giving nonzero probabilities are revealed in Table 5.13. After recalibration, all probabilities are zero.

Table 5.13. Motomco: Selected probabilities of discounting corn that should not be discounted

Motomco	$Pr(M_\tau > 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.3	0.016	0.0
15.4	0.074	0.0
15.5	0.226	0.0

Finally, the results for the Steinlite meter are the most striking, as was previewed by the probability examples at the beginning of this section. In 1979, a country elevator using the Steinlite meter to test for moisture content would discount corn with the moisture content of 14.9 percent to 15.5 percent, which should not be discounted, 100 percent of the time. Corn with as low as 14.6 percent true moisture content had a 50-50 chance of being wrongly discounted. Since recalibration of the meters, these probabilities are all zero. Table 5.14 exhibits these results.

Table 5.14. Steinlite: Selected probabilities of discounting corn that should not be discounted

Steinlite	$Pr(M_\tau > 15.5 \mid T=\tau)$	
τ	1979	1981-82
14.4	0.025	0.0
14.5	0.167	0.0
14.6	0.496	0.0
14.7	0.821	0.0
14.8	0.965	0.0
14.9	0.996	0.0
15.0	1.00	0.0
15.1	1.00	0.0
15.2	1.00	0.0
15.3	1.00	0.0
15.4	1.00	0.0
15.5	1.00	0.0

5.3.2. The probability of not discounting when should

Corn with a true moisture content larger than 15.5 percent does not meet the grade requirement for No. 2 corn. It should be discounted for excessive moisture content, but it will not be if the electronic meter used to measure its moisture content reads a value of 15.5 percent or less. The probability of corn with a true moisture content of $T = \tau$, where $\tau > 15.5$, not being discounted when it should be is

$$Pr(M_\tau \leq 15.5 | T=\tau).$$

The methods described in section 5.2 can be used to compute this probability for relevant values of T.

For example, suppose a sample of corn with a true moisture content of 15.9 percent was tested on a Motomco meter in 1979. This corn should have been discounted for excessive moisture content. But, the probability, determined in equation (5.11), that it was not discounted is

$$Pr(M_{15.9} \leq 15.5 | T=\tau) = 0.027.$$

There was a 2.7 percent chance that this corn would have been treated as meeting the standards.

If this same sample were tested by the same meter after 1981, after recalibration, the probability of not discounting this corn when it should be is

$$Pr(M_{15.9} \leq 15.5 | T=15.9) = 0.999.$$

Now the corn will almost certainly be found to meet the standards and will not be discounted, though, with a true moisture content of 15.9 percent, it should be discounted. Recalibration significantly raised the probability of making the wrong discount decision.

Similar calculations were completed for values of T at every one-tenth of a percentage point between 15.5 percent and 30.0 percent. The resulting probabilities were plotted against true moisture content for each meter and each period. The graphs of the results are displayed in Figures 5.5, 5.6, 5.7, and 5.8, for the Burrows, GAC II, Motomco, and Steinlite meters, respectively. Once again, each figure contains the graphs for both periods to facilitate easy visual comparison.

Like before, let us examine these results more closely for individual meters. Consider first the Burrows meter. In 1979, a load of corn with true moisture content of 15.6 percent or greater, delivered to an elevator for sale, and tested for moisture content by a Burrows meter, would have been correctly discounted for excessive moisture. But since 1981, corn with true moisture content between 15.6 percent and 15.9 percent will almost certainly be found to be acceptable by trade standards, and will not be discounted for excessive moisture, though it should be. In fact, corn with true moisture content as high as 16.1 percent has a probability of 63.5 percent of not being discounted. These results are listed in Table 5.15.

In 1979, the GAC II meter had a probability of only 13.7 percent of reading a value of moisture content within trade standards when the moisture content is actually 15.6 percent, a value that is unacceptable

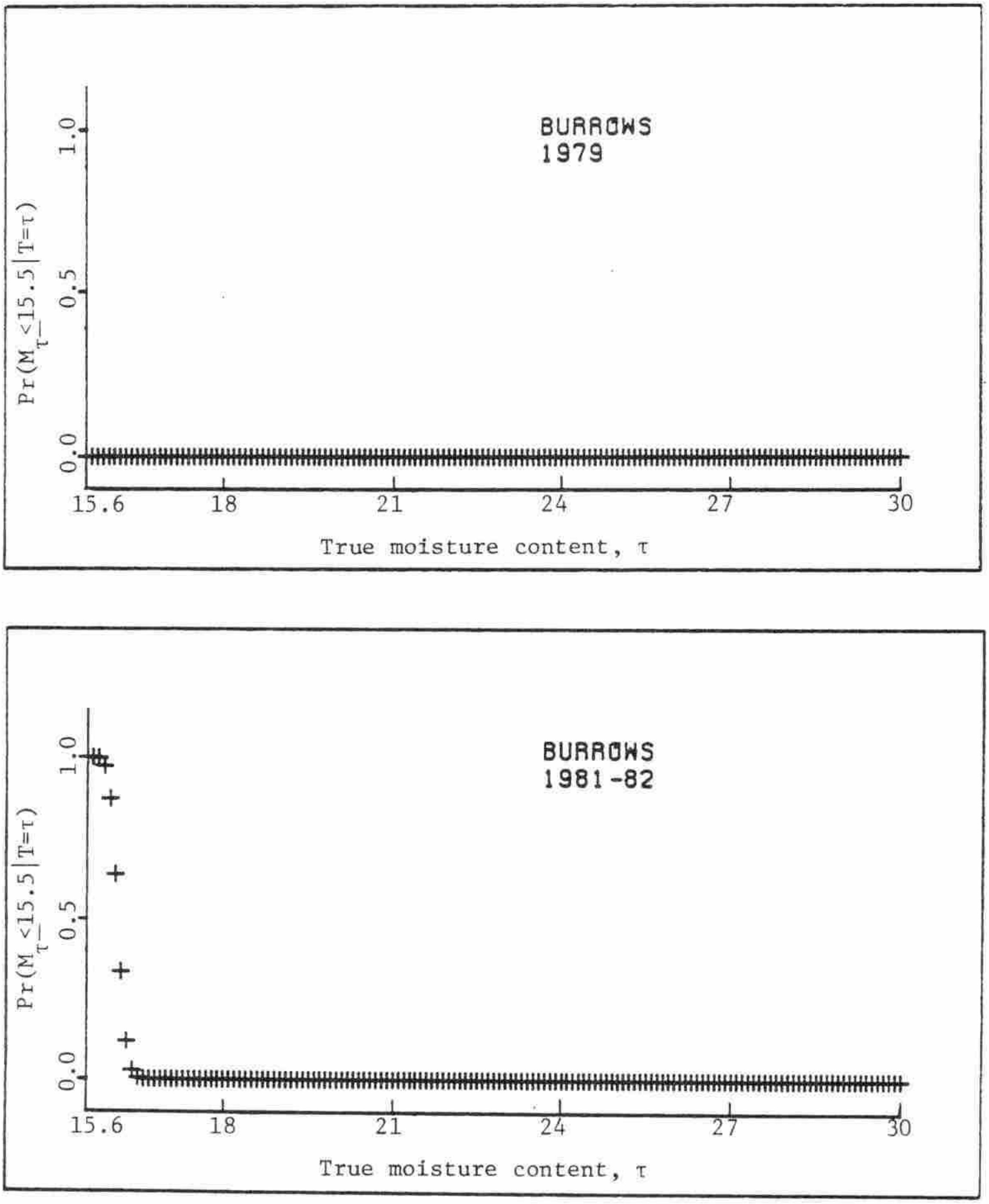
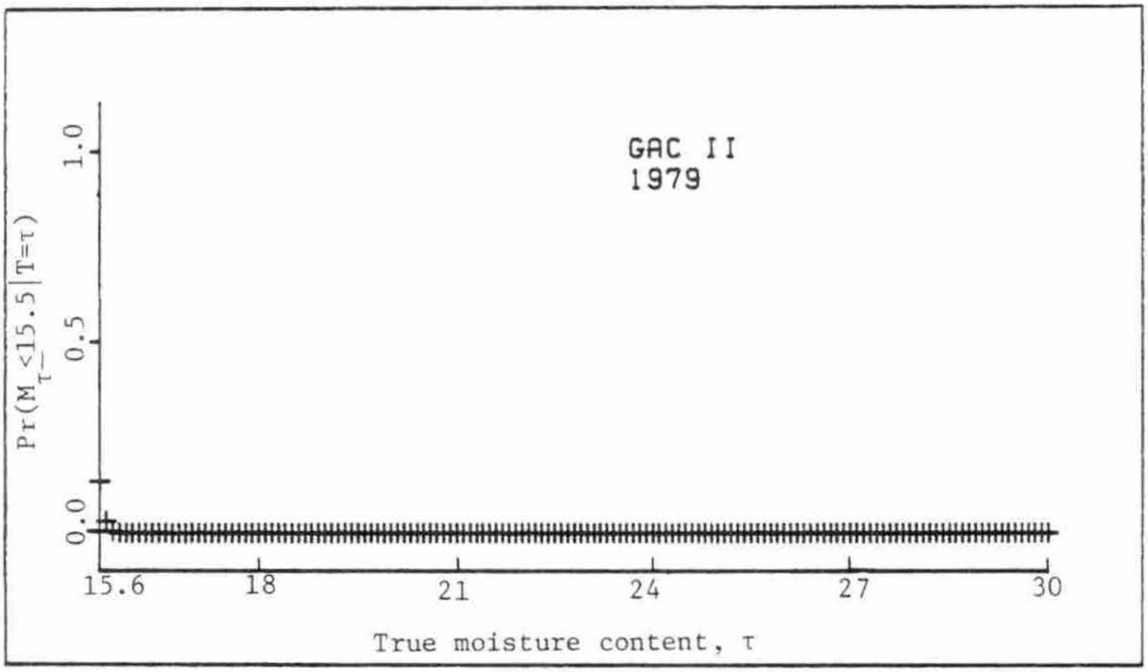


Figure 5.5. Burrows: Probabilities of not discounting corn that should be discounted



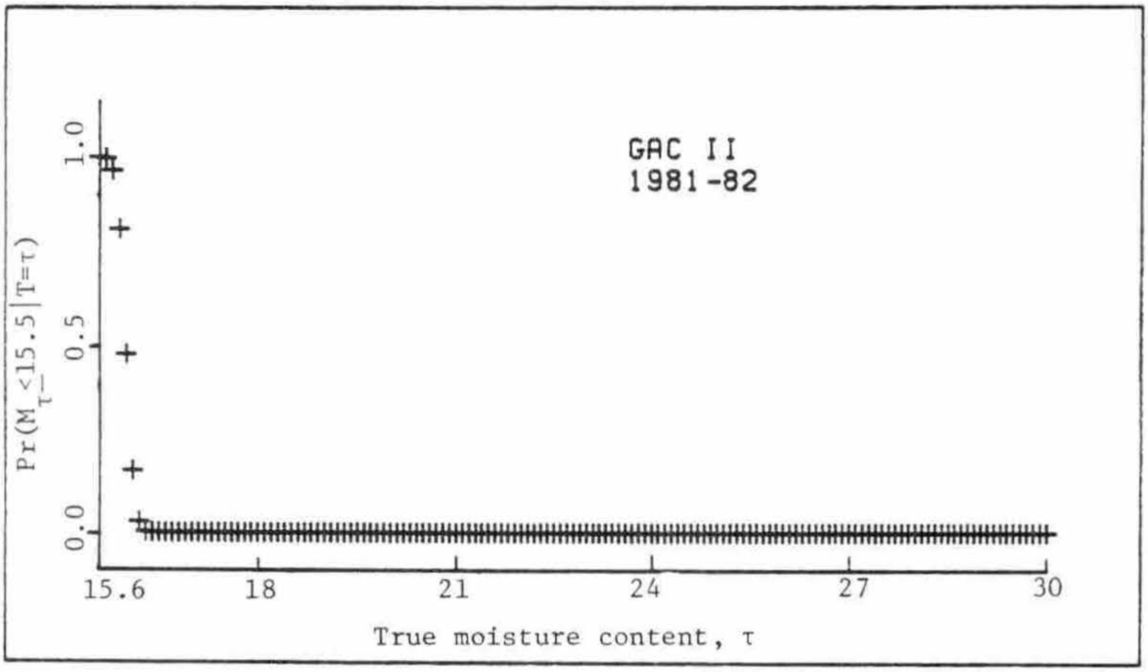


Figure 5.6. GAC II: Probabilities of not discounting corn that should be discounted

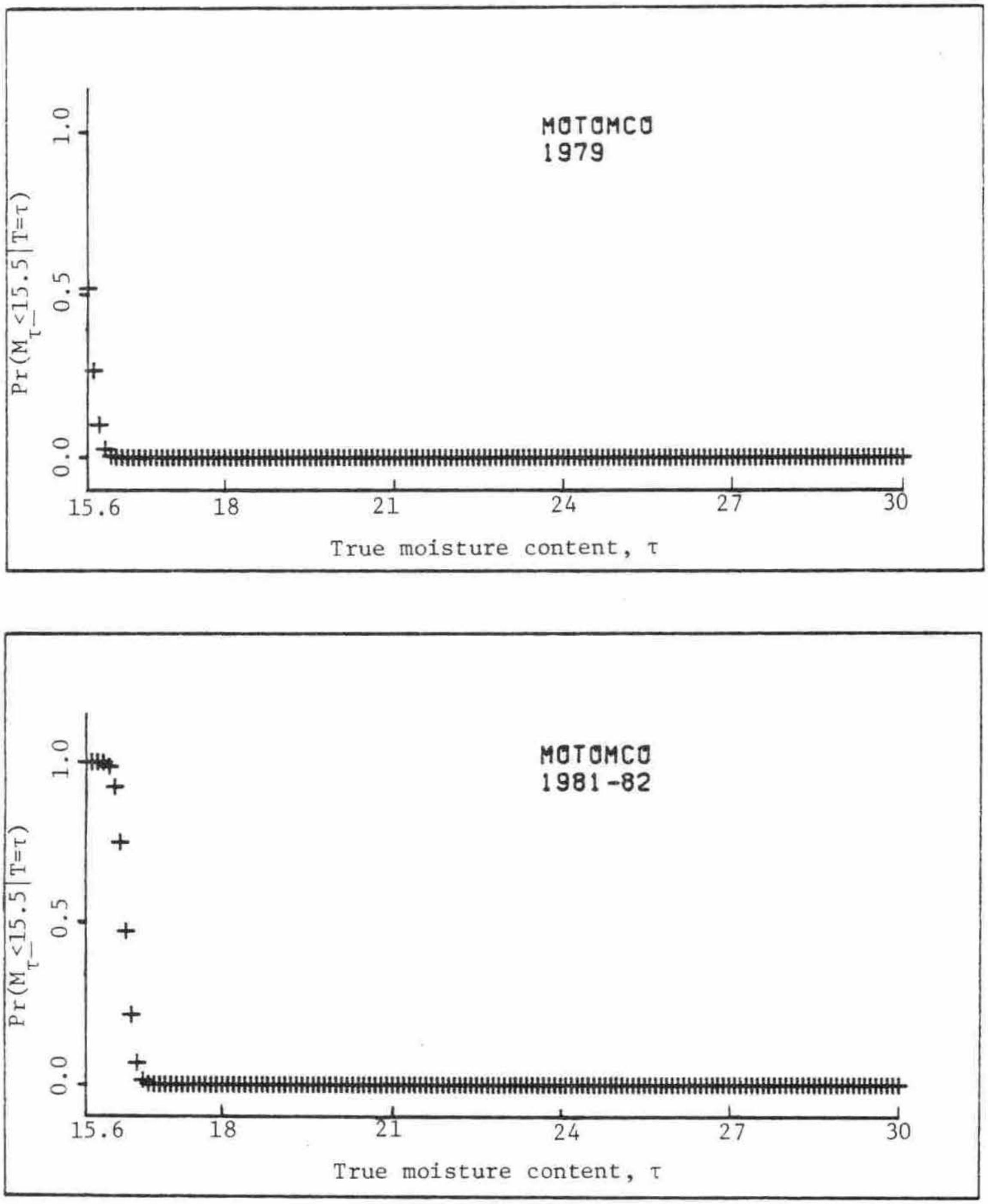


Figure 5.7. Motomco: Probabilities of not discounting corn that should be discounted

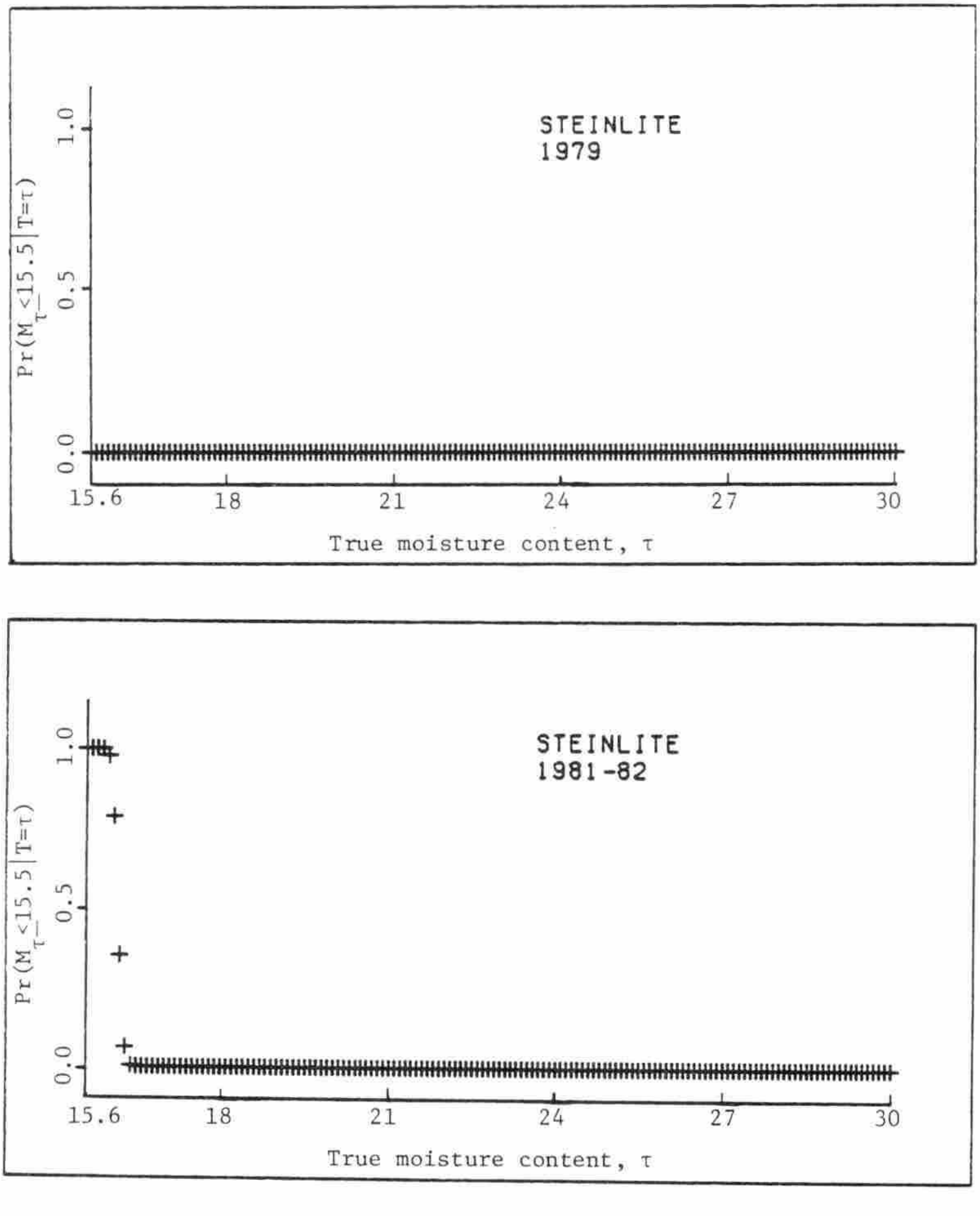


Figure 5.8. Steinlite: Probabilities of not discounting corn that should be discounted

Table 5.15. Burrows: Selected probabilities of not discounting corn that should be discounted

Burrows	$Pr(M_\tau \leq 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.6	0.0	1.00
15.7	0.0	1.00
15.8	0.0	0.997
15.9	0.0	0.973
16.0	0.0	0.871
16.1	0.0	0.635
16.2	0.0	0.335
16.3	0.0	0.119
16.4	0.0	0.027
16.5	0.0	0.0

to the trade. After recalibration, corn with true moisture content of 15.6-15.8 percent has a probability of one or almost one of not being discounted when it should be. Corn with a true moisture content of 16.0 percent, a half percentage point too high, has nearly a 50 percent chance of not being discounted. Table 5.16 exhibits these results.

Table 5.16. GAC II: Selected probabilities of not discounting corn that should be discounted

GAC II	$Pr(M_\tau \leq 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.6	0.137	1.00
15.7	0.031	0.997
15.8	0.0	0.965
15.9	0.0	0.810
16.0	0.0	0.478
16.1	0.0	0.164
16.2	0.0	0.029
16.3	0.0	0.0

In 1979, corn with a true moisture content of 15.6 percent, tested using a Motomco meter, had a probability of slightly greater than 50 percent of not being discounted, though it should be. This is relatively high when compared to other meters for that period. But, now, this probability is one or almost one for corn with true moisture content as high as 16.0 percent. Corn with a true value of 16.3 percent has nearly a 50-50 chance of being allowed to pass moisture testing. Table 5.17 exhibits these results.

Table 5.17. Motomco: Selected probabilities of not discounting corn that should be discounted

Motomco	$Pr(M_\tau \leq 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.6	0.526	1.00
15.7	0.271	1.00
15.8	0.101	1.00
15.9	0.027	0.999
16.0	0.0	0.986
16.1	0.0	0.924
16.2	0.0	0.751
16.3	0.0	0.475
16.4	0.0	0.216
16.5	0.0	0.067
16.6	0.0	0.014
16.7	0.0	0.0

Finally, consider the Steinlite meter. In 1979, corn with true moisture content larger than 15.5 percent, tested on this meter, would have been judged, correctly, to be unacceptable to the trade with respect to moisture content. But, after recalibration, corn with true moisture content as high as 16.0 percent has a probability of one or nearly one of not being discounted when it should be. These probabilities are displayed in Table 5.18.

Table 5.18. Steinlite: Selected probabilities of not discounting corn that should be discounted

Steinlite	$Pr(M_\tau \leq 15.5 \mid T=\tau)$	
τ	1979	1981-82
15.6	0.0	1.00
15.7	0.0	1.00
15.8	0.0	1.00
15.9	0.0	1.00
16.0	0.0	0.977
16.1	0.0	0.787
16.2	0.0	0.352
16.3	0.0	0.063
16.4	0.0	0.0

5.3.3. Probabilities of other types of error

The probabilities of other types of error can be computed using equations (5.7), (5.8), and (5.9), and the methods of sections 4.4 and 5.2. Three specific examples--those described theoretically in section 4.4.2--will be given here.

For instance, the probability that the moisture meter will read a value larger than the true moisture content is

$$Pr(M_\tau > \tau \mid T=\tau),$$

for any value of $T = \tau$. Specific examples follow, using the results presented in Table 5.10.

Suppose we have a sample of corn with a true moisture content of 20.0 percent. In 1979, the probability that the GAC II meter would give a reading of moisture content greater than 20.0 percent was

$$\Pr(M_{20.0}>20.0|T=20.0) = 0.934.$$

This probability was calculated in equation (5.12). That is, there was a 93.4 percent chance that the meter would read a value larger than 20.0 percent, the sample's true moisture content. If this sample of corn had a true moisture content of 26.0 percent, the probability that the GAC II meter would have read a larger value was

$$\Pr(M_{26.0}>26.0|T=26.0) = 0.863.$$

Since 1981-82, after recalibration, this probability is 0.935.

Probabilities of this type were calculated for each meter for each period for values of true moisture content between 10.0 percent and 30.0 percent. The resulting graphs, with probability plotted against true moisture content, are displayed in Figures 5.9, 5.10, 5.11, and 5.12 for the Burrows, GAC II, Motomco, and Steinlite meters, respectively.

In addition, from equation (4.20), we may determine the probability that the size of the measurement error is larger (or smaller) than a specified value for a given $T = \tau$. Here we have computed the probability that the measurement error will be larger than 1.0 percent. Letting measurement error be denoted by $E = M - T$, this is

$$\Pr(E_{\tau}>1.0|T=\tau) = \Pr(M_{\tau}>\tau+1.0|T=\tau).$$

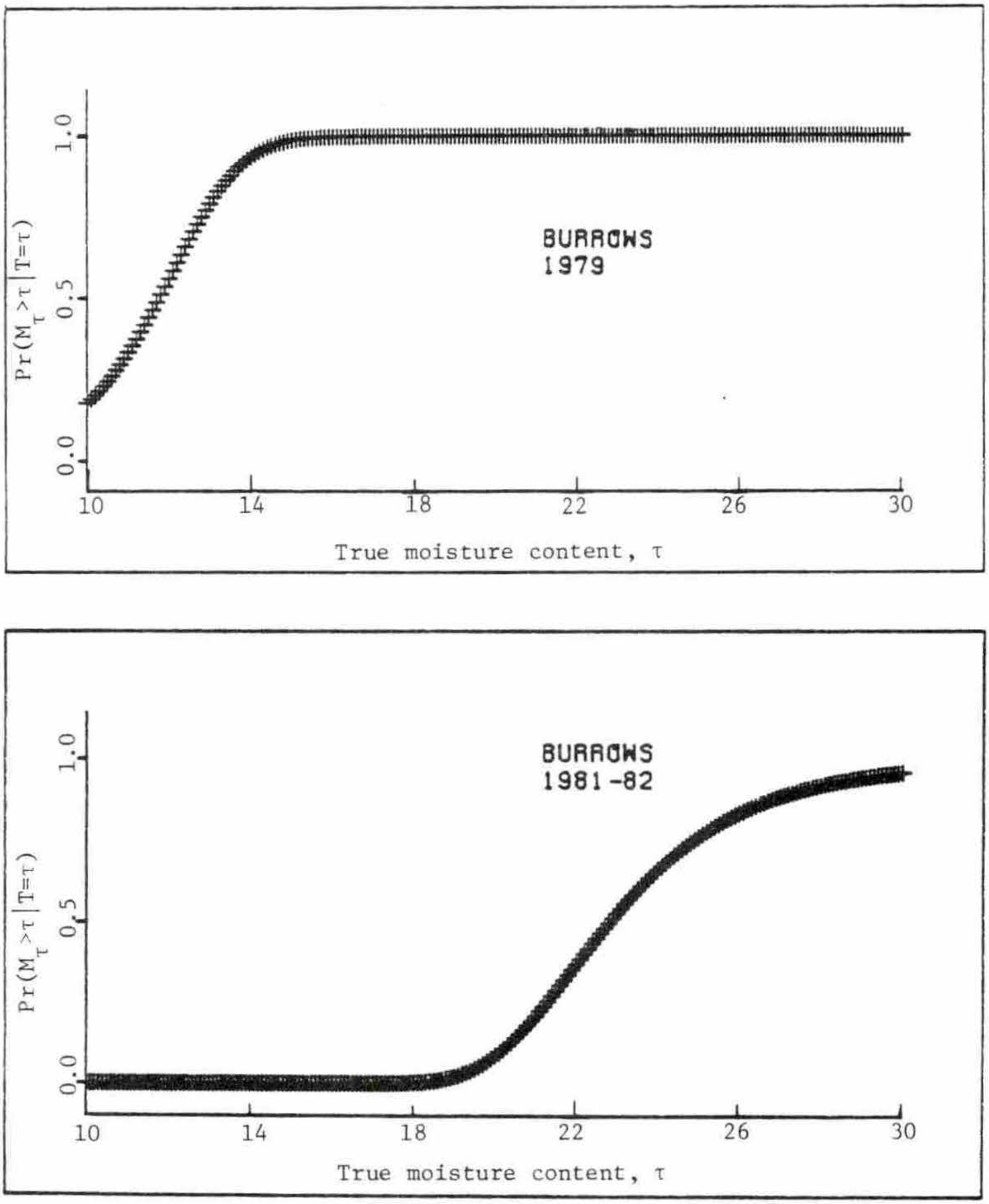


Figure 5.9. Burrows: Probabilities of the meter reading a value larger than the true moisture content

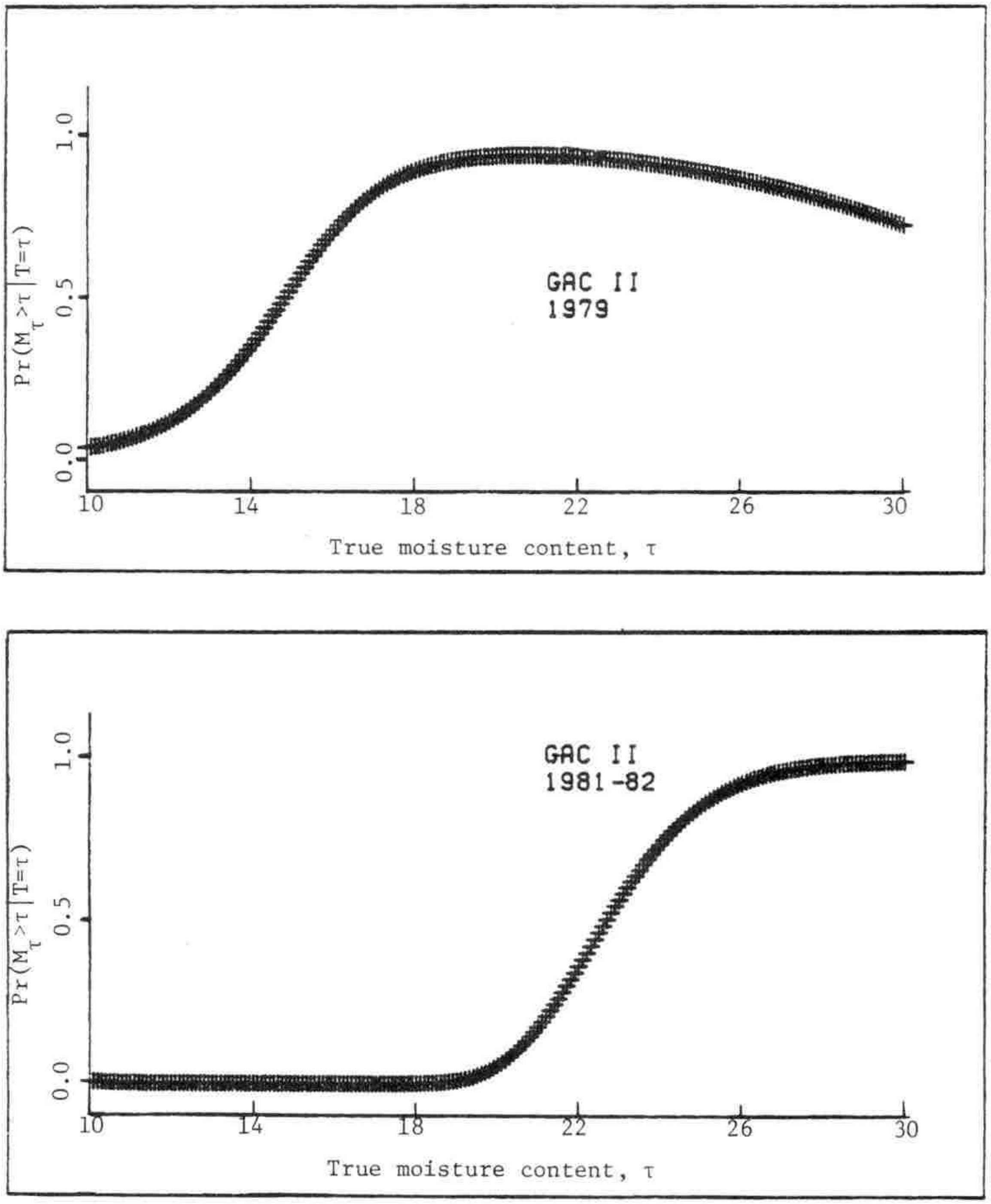


Figure 5.10. GAC II: Probabilities of the meter reading a value larger than the true moisture content

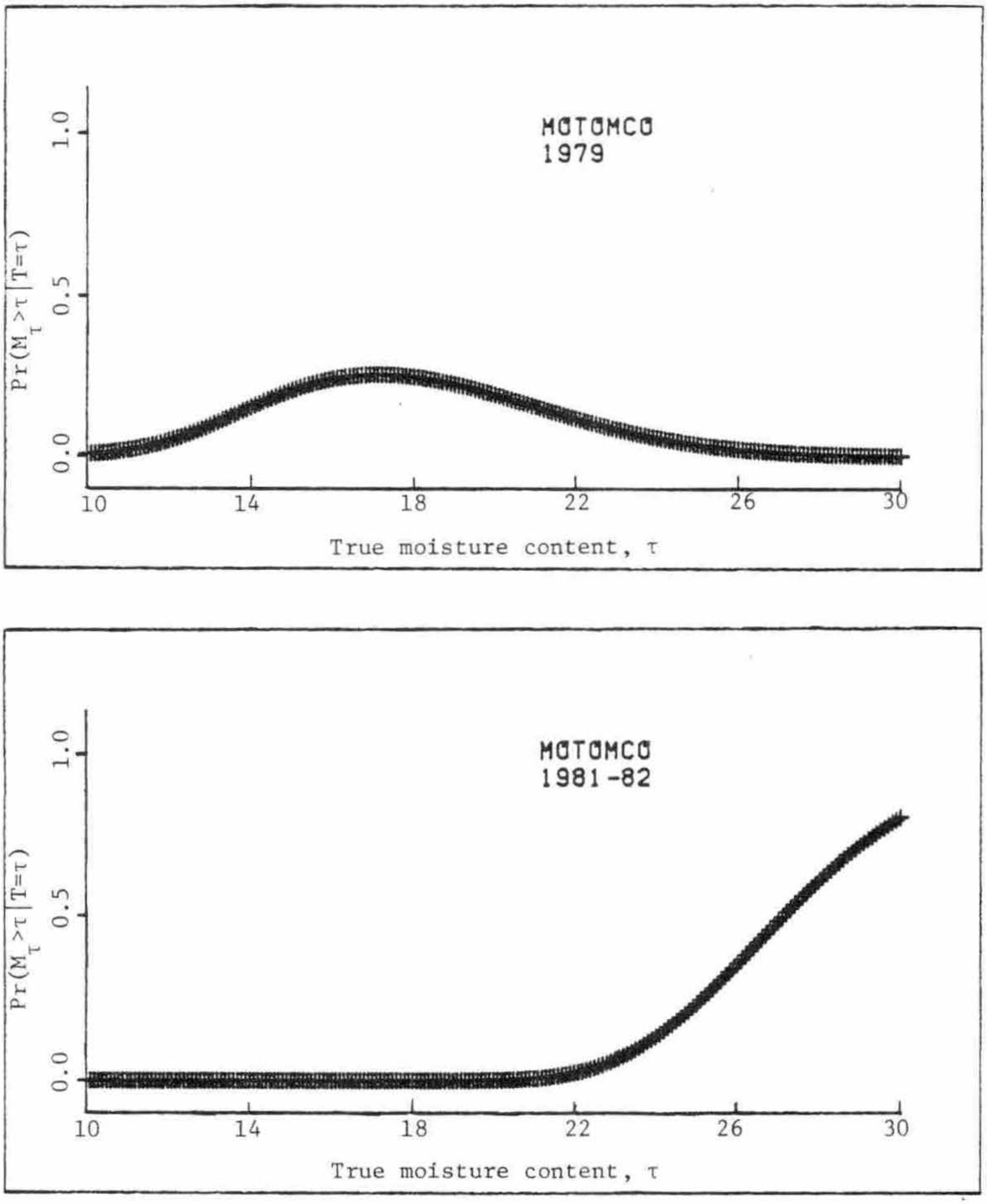


Figure 5.11. Motomco: Probabilities of the meter reading a value larger than the true moisture content

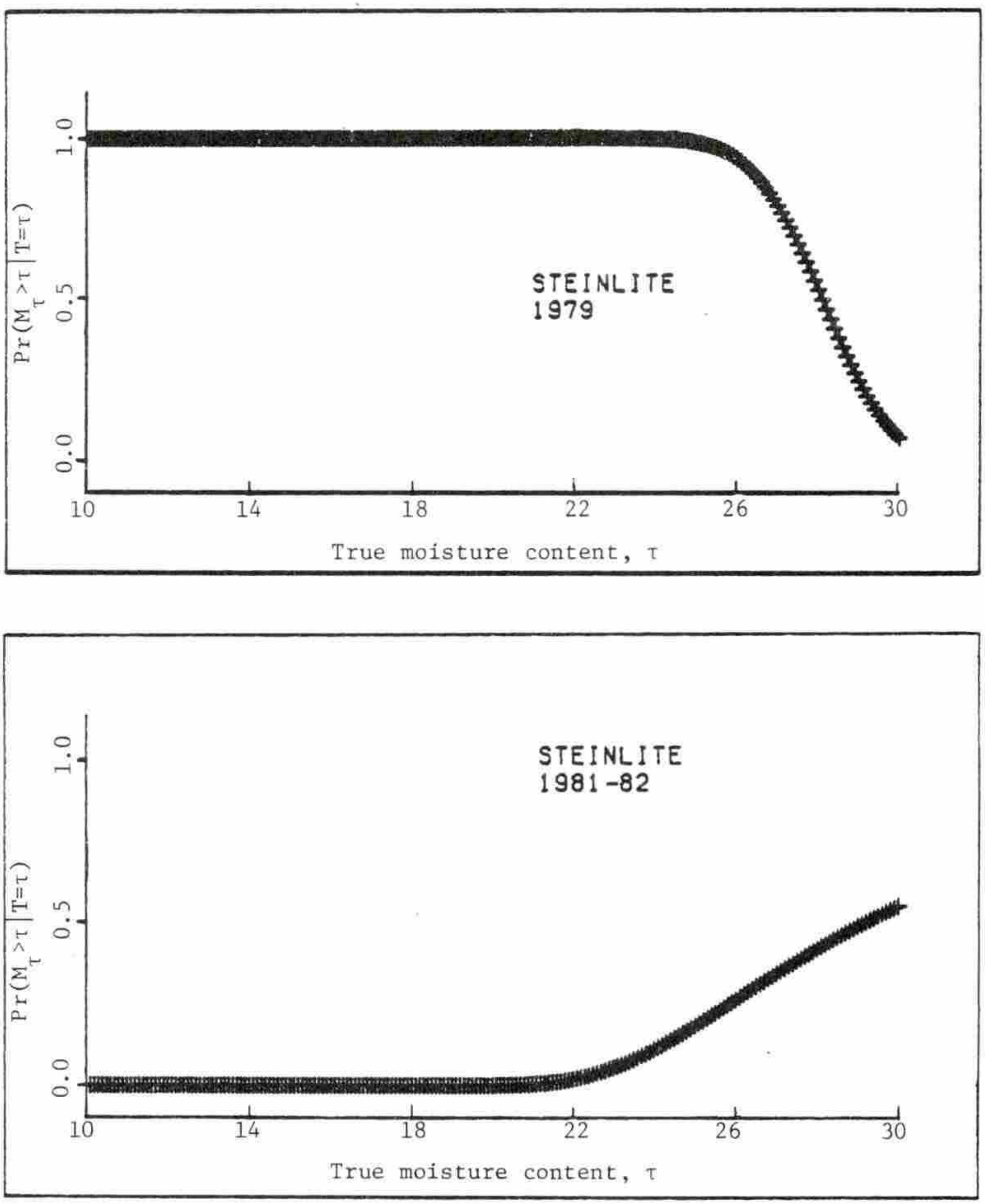


Figure 5.12. Steinlite: Probabilities of the meter reading a value larger than the true moisture content

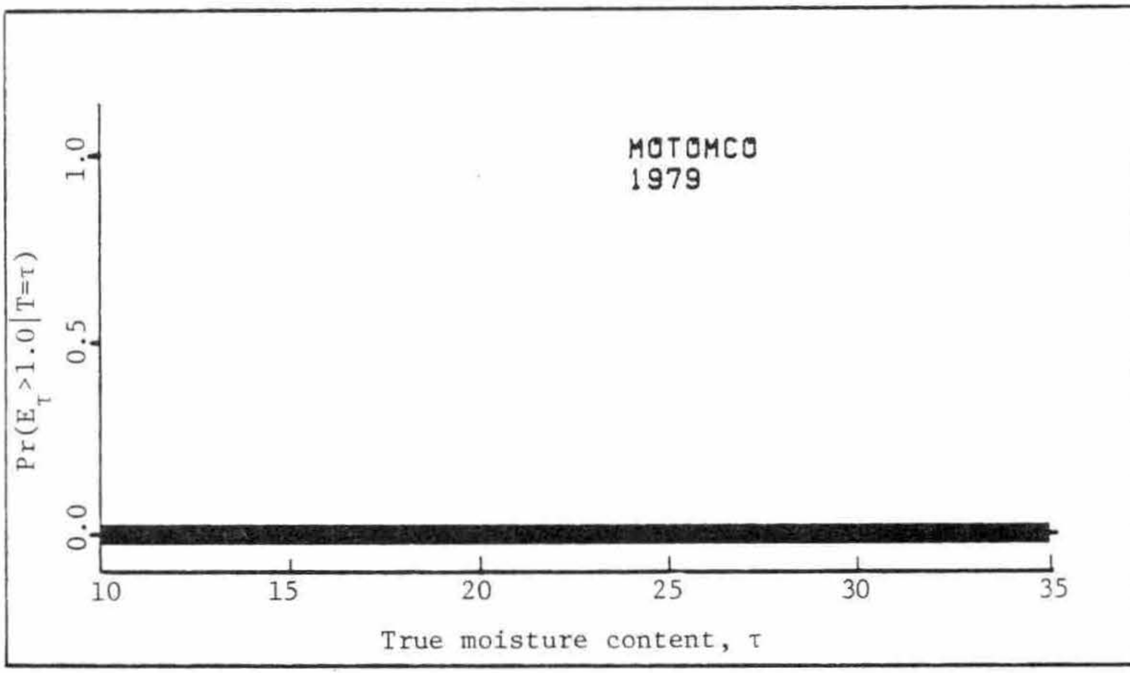
For example, given a sample of corn with true moisture content of 15.5 percent, the probability that the meter measurement error would be greater than 1.0 percent, using the Steinlite meter in 1979, was

$$\begin{aligned} \Pr(E_{15.5}>1.0|T=15.5) &= \Pr(M_{15.5}>16.5|T=15.5) \\ &= 0.487. \end{aligned}$$

Probabilities of this type were calculated for values of T at one-tenth intervals between 10.0 percent and 35.0 percent for the Motomco and Steinlite meters, for each period. The resulting graphs of probability plotted against true moisture content are presented in Figures 5.13 and 5.14, respectively.

The Motomco and Steinlite meters were chosen for the illustrations in the previous and the next examples because of their significance in the grain trade. The Motomco meter is used to determine the moisture content of corn in all trades requiring government inspection, and the Steinlite is the most commonly used meter at country elevators in Iowa.

The final example is somewhat different from the others. In all of the previous examples, true moisture content was the independent variable in the probability calculations, and, thus, on the horizontal axis in the graphs of the results. Now we will fix the true moisture content, let the meter value vary, and ask the question: Given a sample of corn with true moisture content, $T = \tau$, what are the probabilities that the meter will read a value larger than various specified values, m, of M? That is, we are interested in



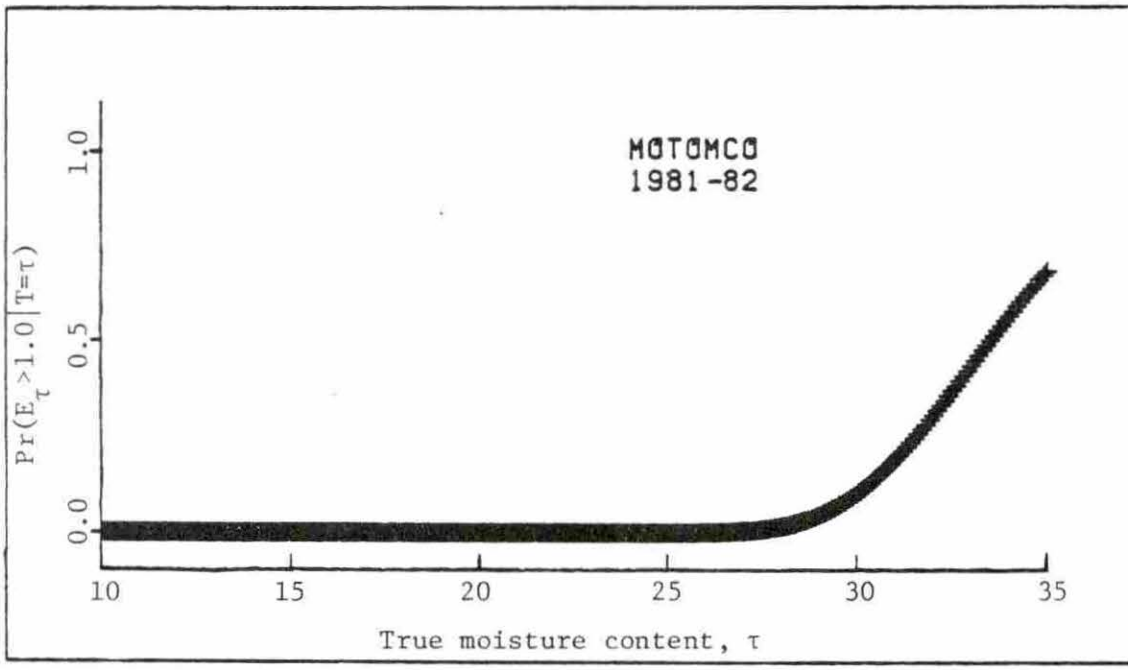


Figure 5.13. Motomco: Probabilities of the measurement error exceeding 1.0 percent

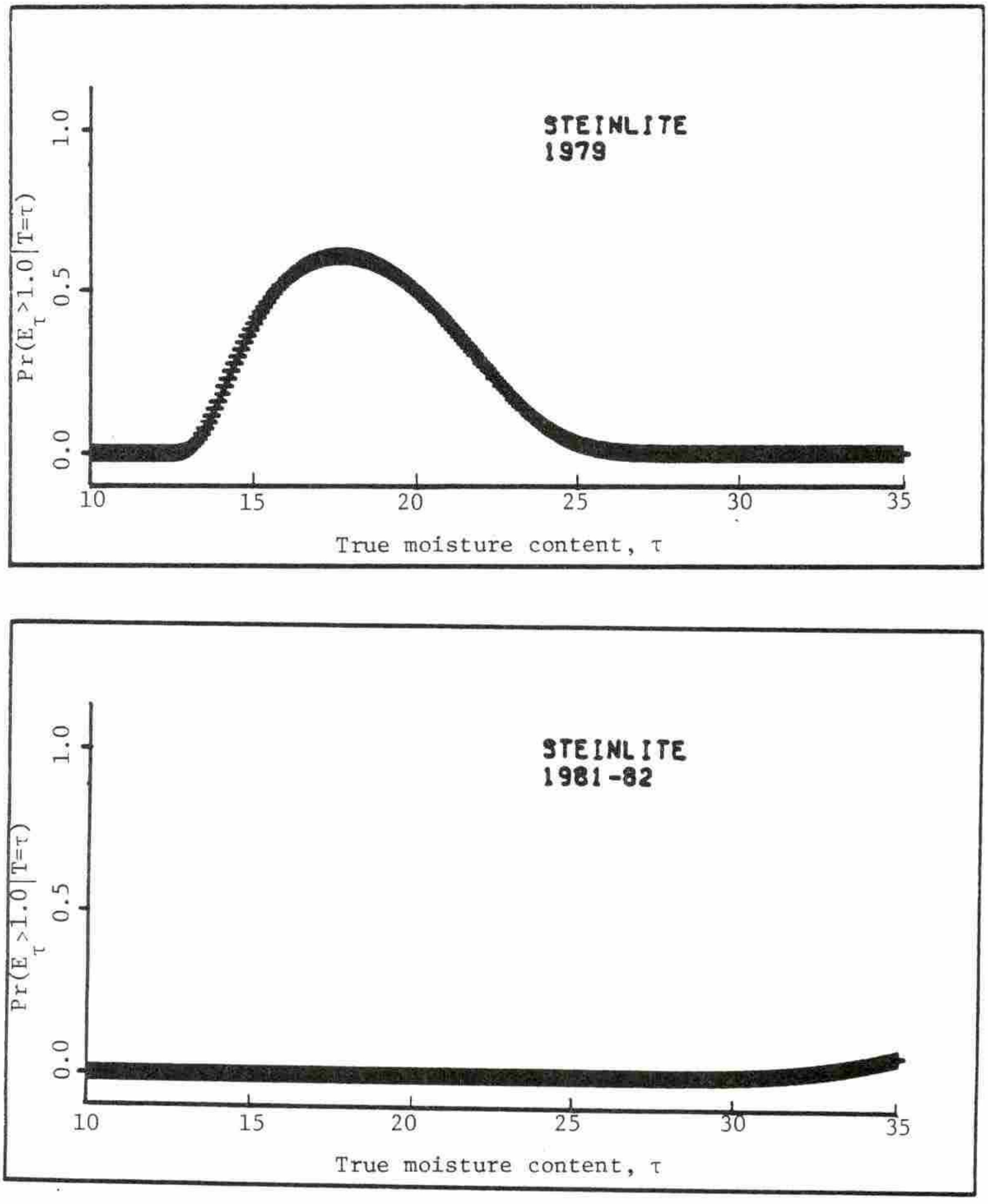


Figure 5.14. Steinlite: Probabilities of the measurement error exceeding 1.0 percent

$$Pr(M_{\tau}>m|T=\tau),$$

where τ is fixed while m varies.

For example, given a sample of corn with a true moisture content of 15.5 percent, the probability that the meter reading will be greater than 16.0 percent is

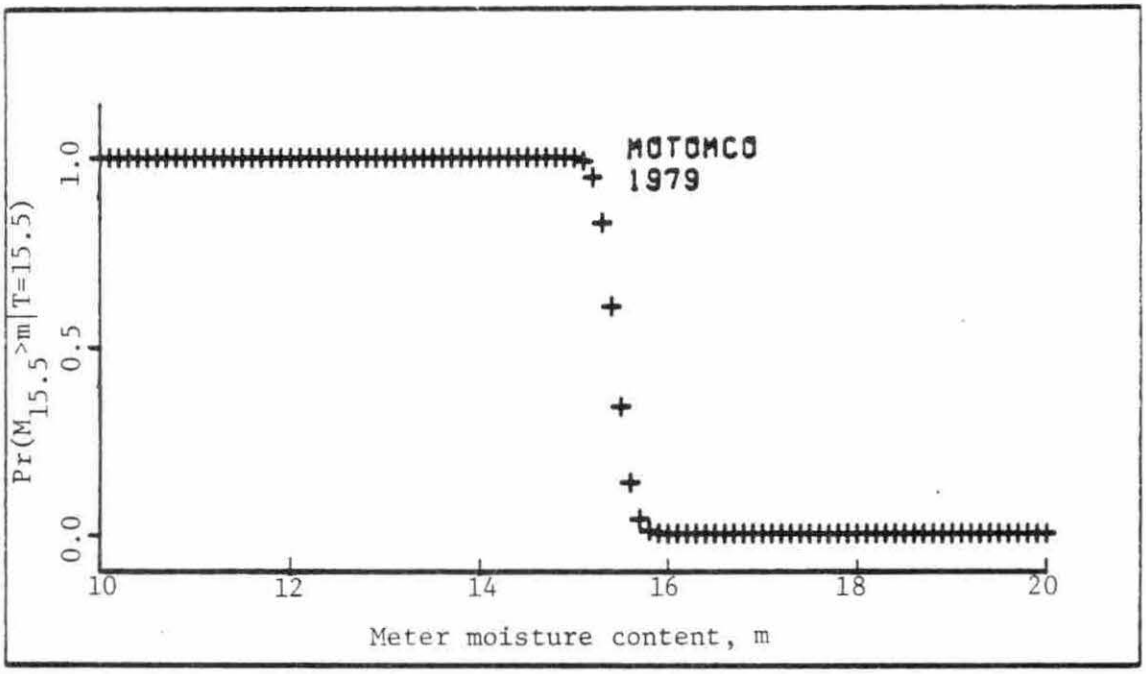
$$Pr(M_{15.5}>16.0|T=15.5).$$

For the Steinlite meter in 1979, this probability was equal to 1.0. Notice that, even though the corn had a true moisture content of only 15.5 percent, the Steinlite meter was sure to give a reading larger than 16.0 percent.

Probabilities of the type

$$Pr(M_{15.5}>m|T=15.5)$$

are computed for values of m between 10.0 percent and 20.0 percent at intervals of one-tenth of a percent. This was done for the Motomco and Steinlite meters for both periods, and the resulting graphs are shown in Figures 5.15 and 5.16, respectively. In order to evaluate the performance of these meters, these graphs should be compared to the one in Figure 4.3, which illustrates the ideal graph of this type of probability.



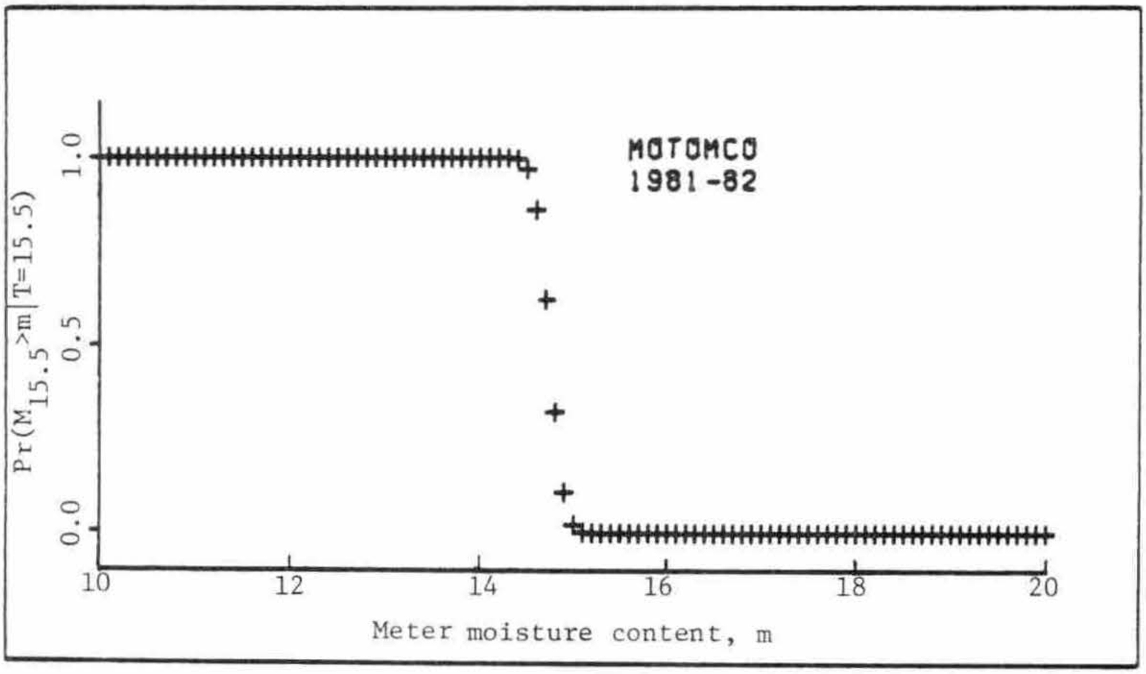


Figure 5.15. Motomco: Probabilities that the meter reading is larger than a specified value when T = 15.5 percent

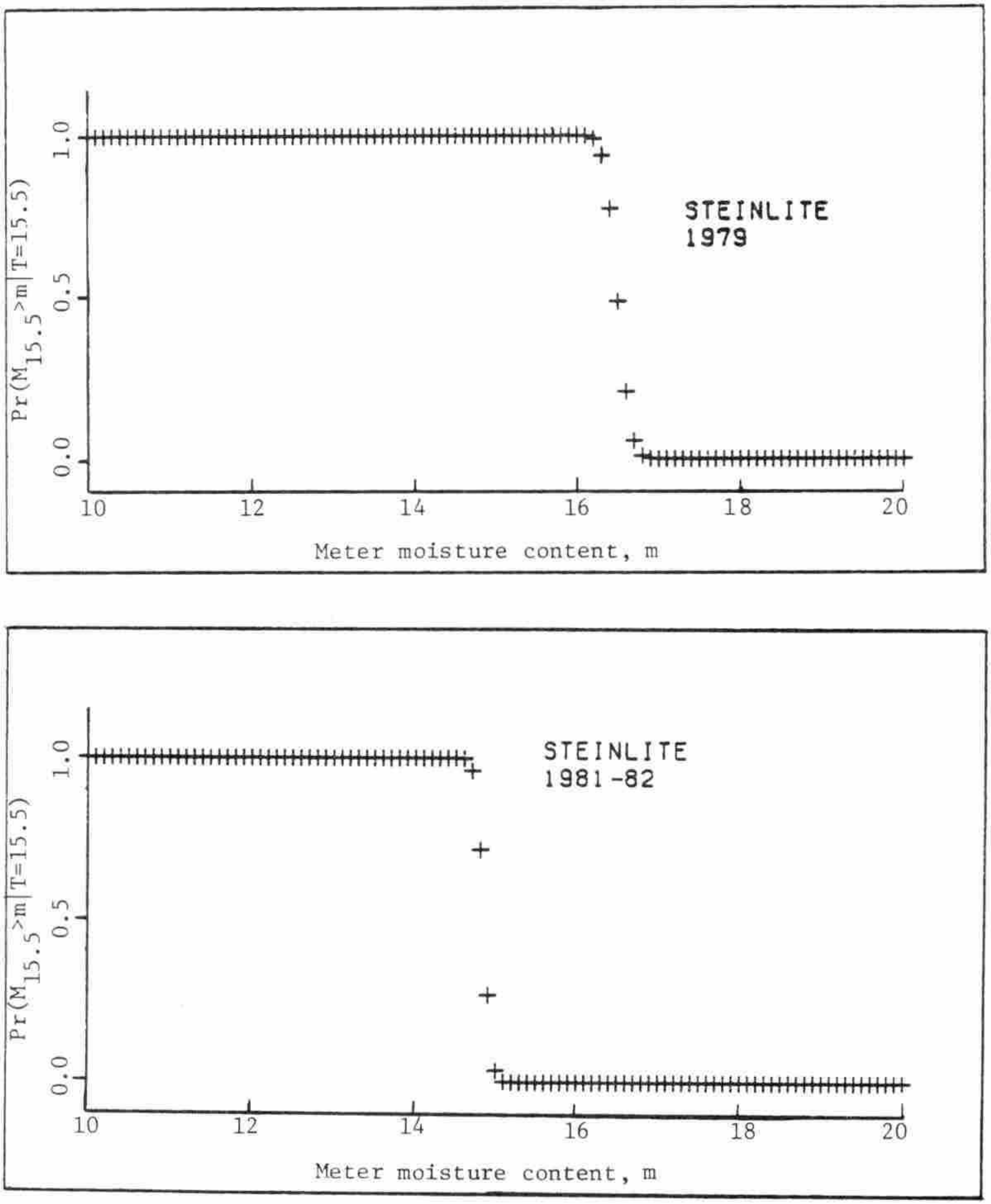


Figure 5.16. Steinlite: Probabilities that the meter reading is larger than a specified value when T = 15.5 percent

6. CONCLUSIONS AND IMPLICATIONS

1) Recalibration of the moisture meters did, indeed, have an effect. The results of the tests for equality of parameters reported in section 5.1.3 support the conclusion that the relationship between meter measurements and true moisture content has changed due to recalibration, for all four brands of meters. In addition, comparison of the graphs of the probability of error with respect to true moisture content, for the types of error considered in section 5.3, lends strong support to the conclusion that recalibration was effective for all four meters. It is unfair to judge, though, that the meters are more accurate now than they were before recalibration. Within certain ranges, they are indeed more accurate; but in other ranges, the accuracy of the meters, in terms of the probability of error, has decreased. Thus, we conclude that there has been marked improvement in the meters at lower moisture levels, but more research must be done in order to achieve greater overall accuracy.

2) The results revealed in Section 5.3.1 verify that, before recalibration, the meters were biased against the farmers. That is, it was very likely that, at the country elevator, corn with true moisture content close to, but less than 15.5 percent, would be treated like corn with moisture content greater than 15.5 percent, and the prices the farmers received for their corn would be discounted. This was especially true if the corn was tested for moisture content using a Steinlite meter. Since recalibration of the meters, this probability, of corn being discounted when it should not be, is equal to zero, for all four brands of meters.

3) The results displayed in section 5.3.2 suggest that the meters are now biased against the country elevators. That is, since recalibration of the meters, the probability is equal to one, or is very close to one, that the meters will read a value less than or equal to 15.5 percent when the corn actually has a true moisture content somewhat higher than 15.5 percent. This corn should be discounted for excessive moisture content, but will not be. Thus, since this corn is not of acceptable moisture quality to meet trade standards, the elevators must condition this corn as necessary and must absorb the costs thereof. This translates into a loss of operating money to the elevators.

4) Consider the variance functions relating the variance of three individual meter readings to the true moisture content. The estimation results appear in section 5.1.1. Recall that all estimated equations were statistically significant at the 0.001 level, but the R^2 values, measuring the proportion of the variation in V explained by the model, were all relatively low. This suggests that the relationship between V, the variance, and T, the moisture content, is significant, but not very strong. That is, there is much randomness in the variability of the moisture meters that cannot be explained by the level of true moisture content.

5) Theoretically, for generalized least squares models with the assumptions set forth in section 4.2.2 and estimated by the methods described there, we have

$$E(MSE) = E(\frac{ESS}{edf}) = \frac{E(ESS)}{edf} = \frac{edf}{edf} = 1,$$

where MSE = mean-squared error, ESS = error sum of squares, and edf = error degrees of freedom, all for the estimated model. But, as can be seen in the results reported in section 5.1.2, the estimate of MSE is quite different from one for all models. It is, in fact, much larger, the smallest being 38.438 (see Table 5.6). This is believed to be due to the poor fit of the variance function, which provided estimated variances used in the GLS estimation. This result also lends support to the conclusion that true moisture content alone is not sufficient in explaining the variability of the moisture meters.

6) Further research into the variance function is recommended. Variability of the meters may be attributable to other factors not yet identified. This is particularly suggested by the results for the Motomco meter. For moisture levels lower than 21.0 percent, the calibration of this meter was left unchanged during the period of recalibration. Yet its probability graphs in section 5.3 are quite different between the two periods. This lends support to the suggestion that meter readings are affected by differences in characteristics of the corn due to differences in growing seasons. Research by the Agricultural Engineering Department at Iowa State University is continuing in an attempt to identify other factors which affect the measurement of moisture content. In addition, other functional forms of the model in terms of true moisture content alone may be relevant. In the process of this study, a functional form of V in terms of the natural logarithm of T was briefly considered. It was found to be promising with regard to the variance function alone, but provided unacceptable results in the GLS estimation stage, in terms of the evaluation criteria used in this study and discussed in section 5.1.2.

7) The major results of this study have been with regard to the discount decision. That is, the primary focus has been on the probability of making the wrong discount decision. Drying charges are also related to moisture content. The results of this study may be extended into consideration of the probable size of measurement error in determining the moisture content, and the impact on the drying charge.

8) Some moisture meters are more accurate than others in certain ranges of moisture content. Thus, it may pay farmers to "shop around" for an elevator using a meter that will give a more favorable moisture reading depending on whether the farmer wishes to sell or store his grain.

9) These results may be used in further study of the economic effects of the recalibration of the moisture meters. It should be noted that meter bias does not necessarily imply pricing bias in the same direction. For instance, the current meter bais in favor of the farmers should not be interpreted as pricing bias in their favor as well. Thus, particular consideration must be given to the prospective effects of more accurate moisture measurement on the price-determining system and the cost structure at country elevators.

10) Finally, one of the major results of this project is the method itself by which these probabilities of error were calculated. This procedure is relatively general, and may be applied in considering 1) the accuracy of moisture meters in other states, 2) the accuracy of moisture meters used on other commodities, or 3) the accuracy of measuring devices of other types, to name a few.

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9. THEORETICAL APPENDIX

The following was developed by G. W. Ladd specifically for the anlaysis required by this project.

9.1. The Theory of the Estimation of $M = g(T)$

Define the following matrices:

$$M = \begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{31} \\ m_{32} \\ m_{33} \\ \vdots \\ m_{i1} \\ m_{i2} \\ m_{i3} \\ \vdots \\ m_{n1} \\ m_{n2} \\ m_{n3} \end{pmatrix}; \text{ when } k = 3,\ T = \begin{pmatrix} 1 & t_1 & t_1^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_3 & t_3^2 \\ \vdots & \vdots & \vdots \\ 1 & t_i & t_i^2 \\ 1 & t_i & t_i^2 \\ 1 & t_i & t_i^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \\ 1 & t_n & t_n^2 \\ 1 & t_n & t_n^2 \end{pmatrix}; \text{ and } U = \begin{pmatrix} u_{11} \\ u_{12} \\ u_{13} \\ u_{21} \\ u_{22} \\ u_{23} \\ u_{31} \\ u_{32} \\ u_{33} \\ \vdots \\ u_{i1} \\ u_{i2} \\ u_{i3} \\ \vdots \\ u_{n1} \\ u_{n2} \\ u_{n3} \end{pmatrix} \quad (9.1)$$

where

M is a 3n x 1 column vector of original individual meter readings, m_{ij};

T is a 3n x k matrix of values of true moisture content;

and

U is a 3n x 1 column vector of stochastic error terms, u_{ij}.

The elements of M and T are the original data. When k = 3, the elements of T are of the form t_i and t_i^2. The subscripts are i = 1, 2, ..., n, and j = 1, 2, 3, where n is the number of reported observations. Thus, 3n is the number of original observations on M.

A model for these data is

$$M = T\beta + U, \text{ where } \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

when a quadratic model (k = 3) is being considered. The following generalized least squares (GLS) assumptions are relevant:

$$1)\ E(U) = 0$$

$$2)\ E(UU') = V$$

where

$$V = \begin{pmatrix} v_1 & & & & & & & & & & & & \\ & v_1 & & & & & & & & & & & \\ & & v_1 & & & & & & 0 & & & & \\ & & & v_2 & & & & & & & & & \\ & & & & v_2 & & & & & & & & \\ & & & & & v_2 & & & & & & & \\ & & & & & & v_3 & & & & & & \\ & & & & & & & v_3 & & & & & \\ & & & & & & & & v_3 & & & & \\ & & & & & & & & & \ddots & & & \\ & & 0 & & & & & & & & v_n & & \\ & & & & & & & & & & & v_n & \\ & & & & & & & & & & & & v_n \end{pmatrix},$$

which is a 3n x 3n diagonal matrix. Each v_i is the true variance associated with the mean of three individual meter readings on the same subsample. Notice that assumption (2) is the assumption of heteroscedasticity in the individual meter readings.

According to the theory of GLS, the best linear unbiased estimate of β is

$$\hat{b} = (T'V^{-1}T)^{-1} (T'V^{-1}M),$$

where

$$(T'V^{-1}T)^{-1} = 1/3 \begin{pmatrix} \sum_i v_i^{-1} & \sum_i t_i v_i^{-1} & \sum_i t_i^2 v_i^{-1} \\ & \sum_i t_i^2 v_i^{-1} & \sum_i t_i^3 v_i^{-1} \\ \text{Symmetric} & & \sum_i t_i^4 v_i^{-1} \end{pmatrix}^{-1}, \tag{9.2}$$

$i = 1, \ldots, n$, and

$$(T'V^{-1}M) = \begin{pmatrix} \sum_j \sum_i m_{ij} v_i^{-1} \\ \sum_j \sum_i m_{ij} t_i v_i^{-1} \\ \sum_j \sum_i m_{ij} t_i^2 v_i^{-1} \end{pmatrix}, \; i = 1, \ldots, n, \text{ and } j = 1, 2\ 3. \tag{9.3}$$

In addition, we can obtain the following:

1) The variance-covariance, or dispersion, matrix of $\hat{b}$ is

$$\mathrm{var}(\hat{b}) = D(\hat{b}) = (T'V^{-1}T)^{-1}.$$

2) The error, or residual, sum of squares is

$$\begin{aligned} ESS(\hat{b}) &= M'V^{-1}M - (M'V^{-1}T)\hat{b} \\ &= \sum_j \sum_i m_{ij}^2 v_i^{-1} - \left(\sum_j \sum_i m_{ij} v_i^{-1},\ \sum_j \sum_i m_{ij} t_i v_i^{-1},\ \sum_j \sum_i m_{ij} t_i^2 v_i^{-1}\right)\hat{b} \end{aligned}$$

where $i = 1, \ldots, n$ and $j = 1, 2, 3$.

3) $E[ESS(\hat{b})] = 3n - 3 = 3(n-1)$.

We do not have the original observations on the meter readings required for the above estimation. We do have reported observations on mean meter readings calculated as

$$\overline{m}_i = 1/3 \sum_{j=1}^{3} m_{ij}.$$

We now have the following matrices:

$$\overline{M} = \begin{pmatrix} \overline{m}_1 \\ \overline{m}_2 \\ \overline{m}_3 \\ \vdots \\ \overline{m}_i \\ \vdots \\ \overline{m}_n \end{pmatrix} ; \text{ when } k = 3,\ \overline{T} = \begin{pmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ \vdots & \vdots & \vdots \\ 1 & t_i & t_i^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \end{pmatrix} ; \text{ and } \overline{U} = \begin{pmatrix} \overline{u}_1 \\ \overline{u}_2 \\ \overline{u}_3 \\ \vdots \\ \overline{u}_i \\ \vdots \\ \overline{u}_n \end{pmatrix} \quad (9.4)$$

where

$$\overline{M} = GM,$$

$$\overline{T} = GT,$$

and

$$\overline{U} = GU.$$

G is an n x 3n grouping matrix defined as

$$G = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \cdots & & & & & & & & & \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & \cdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & & & & \\ & \vdots & & & & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & & & & & & & & \cdots & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

Note that

$$\overline{m}_i = \sum_{j=1}^{3} 1/3m_{ij} = 1/3 \sum_{j=1}^{3} m_{ij},$$

which gives the reported mean meter reading as necessary. Also, the i-th elements of $\overline{T}$ are

$$\overline{t}_i = 1/3t_i + 1/3t_i + 1/3t_i = 1/3(3t_i) = t_i$$

and
$$\overline{t}_i^2 = 1/3t_i^2 + 1/3t_i^2 + 1/3t_i^2 = 1/3(3t_i^2) = t_i^2.$$

The data denoted in matrix form in equation (9.4) will be referred to as the grouped observations or the grouped data.

The model associated with the grouped data is

$$\overline{M} = \overline{T}\beta + \overline{U}, \tag{9.5}$$

which is equivalent to

$$GM = GT\beta + GU.$$

The assumptions are:

1) $E(\overline{U}) = 0$.

2) $E(\overline{UU'}) = E(GUU'G') = G(E(UU'))G'$

$$= GVG' = 1/3 \begin{pmatrix} v_1 & & & & \\ & v_2 & & 0 & \\ & & v_3 & & \\ & 0 & & \ddots & \\ & & & & v_n \end{pmatrix},$$

where GVG' is an n x n diagonal matrix of variances. This is the heteroscedasticity assumption for the grouped data. Note that

$$(GVG')^{-1} = 3 \begin{pmatrix} v_1^{-1} & & & & \\ & v_2^{-1} & & 0 & \\ & & v_3^{-1} & & \\ & 0 & & \ddots & \\ & & & & v_n^{-1} \end{pmatrix}.$$

The GLS estimate of β for the model describing the grouped data is

$$\hat{b}^G = (\overline{T}'(GVG')^{-1}\overline{T})^{-1} (\overline{T}'(GVG')^{-1}\overline{M}),$$

where

$$(\overline{T}(GVG')^{-1}\overline{T})^{-1} = 1/3 \begin{pmatrix} \sum_i v_i^{-1} & \sum_i t_i v_i^{-1} & \sum_i t_i^2 v_i^{-1} \\ & \sum_i t_i^2 v_i^{-1} & \sum_i t_i^3 v_i^{-1} \\ \text{Symmetric} & & \sum_i t_i^4 v_i^{-1} \end{pmatrix}^{-1} \qquad (9.6)$$

$$= (T'V^{-1}T)^{-1} \text{ from equation (9.2)},$$

and

$$(\overline{T}'(GVG')^{-1}\overline{M}) = 3\begin{pmatrix} \sum_i \overline{m}_i v_i^{-1} \\ \\ \sum_i \overline{m}_i t_i v_i^{-1} \\ \\ \sum_i \overline{m}_i t_i^2 v_i^{-1} \end{pmatrix} = \begin{pmatrix} \sum_j\sum_i m_{ij} v_i^{-1} \\ \\ \sum_j\sum_i m_{ij} t_i v_i^{-1} \\ \\ \sum_j\sum_i m_{ij} t_i^2 v_i^{-1} \end{pmatrix} = T'V^{-1}M$$

from equation (9.3). Thus,

$$\hat{b} = \hat{b}^G \qquad (9.7)$$

where $\hat{b}$ contains the parameter estimates for the original data and $\hat{b}^G$ contains the parameter estimates for the grouped data. Furthermore, from equation (9.6), we have the following result concerning the dispersion matrix for $\hat{b}^G$:

$$D(\hat{b}^G) = (\overline{T}(GVG')^{-1}\overline{T})^{-1} = (T'V^{-1}T) = D(\hat{b}).$$

Thus, we can use the reported grouped observations to compute $\hat{b}$ and $D(\hat{b})$ for the estimated relationship between <u>individual</u> meter readings and true moisture content.

Finally, the error sum of squares associated with the model in equation (9.5), describing the grouped data, is

$$ESS(\hat{b}^G) = \overline{M}'(GVG')^{-1}\overline{M} - \overline{M}'(GVG')^{-1}\overline{T}\hat{b}^G$$

$$= 3[\sum_i \overline{m}_i^2 v_i^{-1} - (\sum_i \overline{m}_i v_i^{-1}, \sum_i \overline{m}_i t_i v_i^{-1}, \sum_i \overline{m}_i t_i^2 v_i^{-1})\hat{b}^G].$$

Using the following identities and results:

$$v_i^s = 1/3 \sum_{j=1}^{3} (m_{ij} - \bar{m}_i)^2, \qquad (9.8)$$

$$\bar{m}_i = 1/3 \sum_{j=1}^{3} m_{ij},$$

$$\bar{m}_i^2 = (1/3 \sum_{j=1}^{3} m_{ij})^2,$$

and

$$v_i^s = 1/3[(\sum_{j=1}^{3} m_{ij}^2) - 3\bar{m}_i^2],$$

it can be shown that

$$ESS(\hat{b}^G) = ESS(\hat{b}) - 3 \sum_{i=1}^{3} v_i^s v_i^{-1}, \qquad (9.9)$$

where v_i^s is the i-th sample variance of three meter readings, as defined in equation (9.8). Note that, as a true variance, v_i is unknown. But an estimate of v_i is $\hat{v}_i^s$, which is obtained from the estimated variance function in section 4.2.1. Thus, the elements of the last term in equation (9.9) are of the form $(v_i^s)(\hat{v}_i^s)^{-1}$. But

$$v_i^s = \hat{v}_i^s + \hat{w}_i,$$

where $\hat{w}_i$ is the resulting estimated residual. Thus,

$$\frac{v_i^s}{\hat{v}_i^s} = \frac{\hat{v}_i^s + \hat{w}_i}{\hat{v}_i^s} = 1 + \frac{\hat{w}_i}{\hat{v}_i^s}. \qquad (9.10)$$

Since our samples are sufficiently large so that large sample theory applies, we may take the probability limit of each side of equation (9.10) as follows:

$$\operatorname{plim} \frac{v_i^s}{\hat{v}_i^s} = 1 + (\operatorname{plim} \hat{w}_i)\ (\operatorname{plim} (\hat{v}_i^s)^{-1})$$

$$= 1$$

since $\operatorname{plim} \hat{w}_i = 0$. Therefore,

$$\sum_{i=1}^{n} v_i^s(\hat{v}_i^s)^{-1} = \sum_{i=1}^{n} \operatorname{plim}[v_i^s(\hat{v}_i^s)^{-1}] = \sum_{i=1}^{n} 1 = n,$$

and equation (9.9) can be written as

$$\mathrm{ESS}(\hat{b}^G) = \mathrm{ESS}(\hat{b}) - 3n. \qquad (9.11)$$

9.2. The F-test for Equality of Parameters

In order to derive the F-statistic used in section 4.3 for testing the equality of parameters, define the following models and estimation results for the original data:

For period 1, the 1979 data, we have

$$M_1 = T_1\beta_1 + U_1;$$

$$\hat{b}_1 = (T_1'V_1^{-1}T_1)^{-1}\ (T_1'V_1^{-1}M_1);$$

$$\mathrm{ESS}(\hat{b}_1) = M_1'V_1^{-1}M_1 - M_1'V_1^{-1}T_1\hat{b}_1.$$

For period 2, the pooled 1981 and 1982 data, we have

$$M_2 = T_2\beta_2 + U_2;$$

$$\hat{b}_2 = (T_2'V_2^{-1}T_2)^{-1}\ (T_2'V_2^{-1}M_2);$$

$$ESS(\hat{b}_2) = M_2'V_2^{-1}M_2 - M_2'V_2^{-1}T_2\hat{b}_2.$$

Note that the matrices M_h, T_h, and U_h, $h = 1, 2$, are of the form in equation (9.1).

The null hypothesis to be tested is $H_0 : \beta_1 = \beta_2$, against the alternative $H_A : \beta_1 \neq \beta_2$. The test statistic is an F-ratio:

$$F = \frac{\text{Num}}{\text{Denom}},$$

where

$$\text{Num} = (\hat{b}_1-\hat{b}_2)'[(T_1'V_1^{-1}T_1)^{-1} + (T_2'V_2^{-1}T_2)^{-1}]^{-1}\ (\hat{b}_1-\hat{b}_2)/k$$

and

$$\text{Denom} = [ESS(\hat{b}_1) + ESS(\hat{b}_2)]/[3(n_1+n_2) - 2k].$$

This F-statistic possesses an F distribution with k and $3(n_1+n_2) - 2k$ degrees of freedom. Each n_h is the number of reported observations in period h, h = 1, 2, and k is the number of parameters being estimated by the model. β_h and $\hat{b}_h$ are k x 1.

From equations (9.6) and (9.7), Num can be computed as follows:

$$\text{Num} = (\hat{b}_1^G-\hat{b}_2^G)'\{[\overline{T}_1'(G_1V_1G_1')^{-1}T_1]^{-1} + [T_2'(G_2V_2G_2')^{-1}T_2]^{-1}\}^{-1}\ (\hat{b}_1^G-\hat{b}_2^G)/k.$$

From equation (9.11), we have for Denom:

$$\text{Denom} = [\text{ESS}(\hat{b}_1^G) + \text{ESS}(\hat{b}_2^G) + 3(n_1+n_2)]/[3(n_1+n_2) - 2k].$$

Since k is very small relative to $n_1 + n_2$ for our data and models, Denom can be computed as

$$\text{Denom} = \frac{\text{ESS}(\hat{b}_1^G)+\text{ESS}(\hat{b}_2^G)}{3(n_1+n_2)-2k} + 1.$$

Thus, the results from the estimation using the grouped data can be used to test the equality of the parameters between periods for models describing individual meter readings.